



Semiconductor Devices

# Chapter 8

## The pn Junction Diode

오세용, Ph.D.  
한양대 ERICA, 조교수



# Ideal PN Junction Diode Analysis: Assumption

- 1. Non-degenerately doped step junction, depletion approximation, neutral other than depletion region
- 2. Boltzmann approximation
- 3. Low-level injection condition
- 4. Total current is constant within PN junction diode.

Electron and Hole current are continuous.

Recombination and Generation are negligible in depletion region.

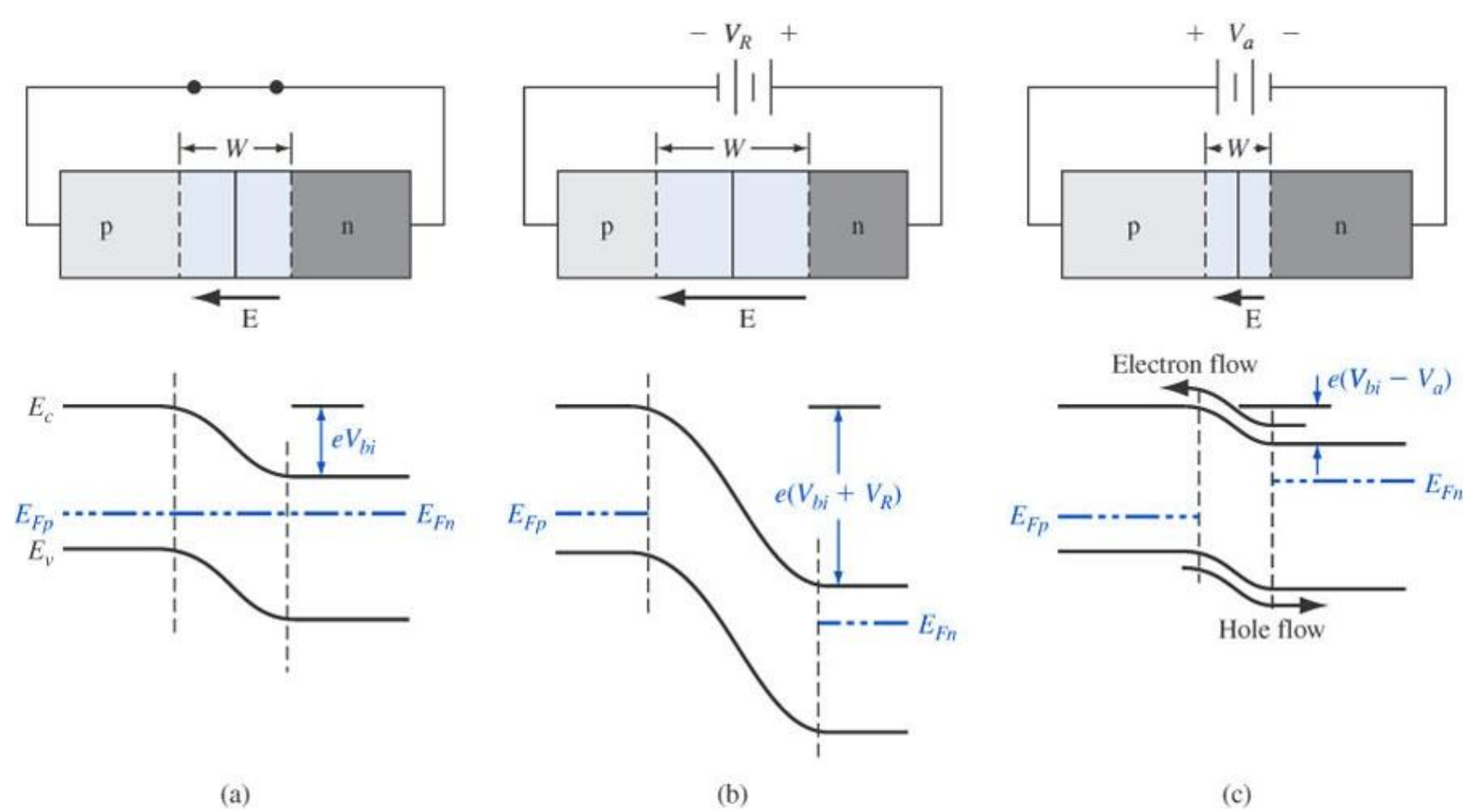
Steady-state condition

## Continuity Equation

$$\frac{\partial p(t, x)}{\partial t} = -\frac{1}{q} \frac{\partial J_p}{\partial x} + g_p' - R_p' \quad R_p' = \frac{\delta p}{\tau_p}$$
$$\frac{\partial n(t, x)}{\partial t} = +\frac{1}{q} \frac{\partial J_n}{\partial x} + g_n' - R_n' \quad R_n' = \frac{\delta n}{\tau_n}$$

$$J_p(t, x) = qp(t, x)\mu_p E(x) - qD_p \frac{\partial p(t, x)}{\partial x}$$
$$J_n(t, x) = qn(t, x)\mu_n E(x) + qD_n \frac{\partial n(t, x)}{\partial x}$$

# PN Junction and Energy Band Diagram under Voltage Bias



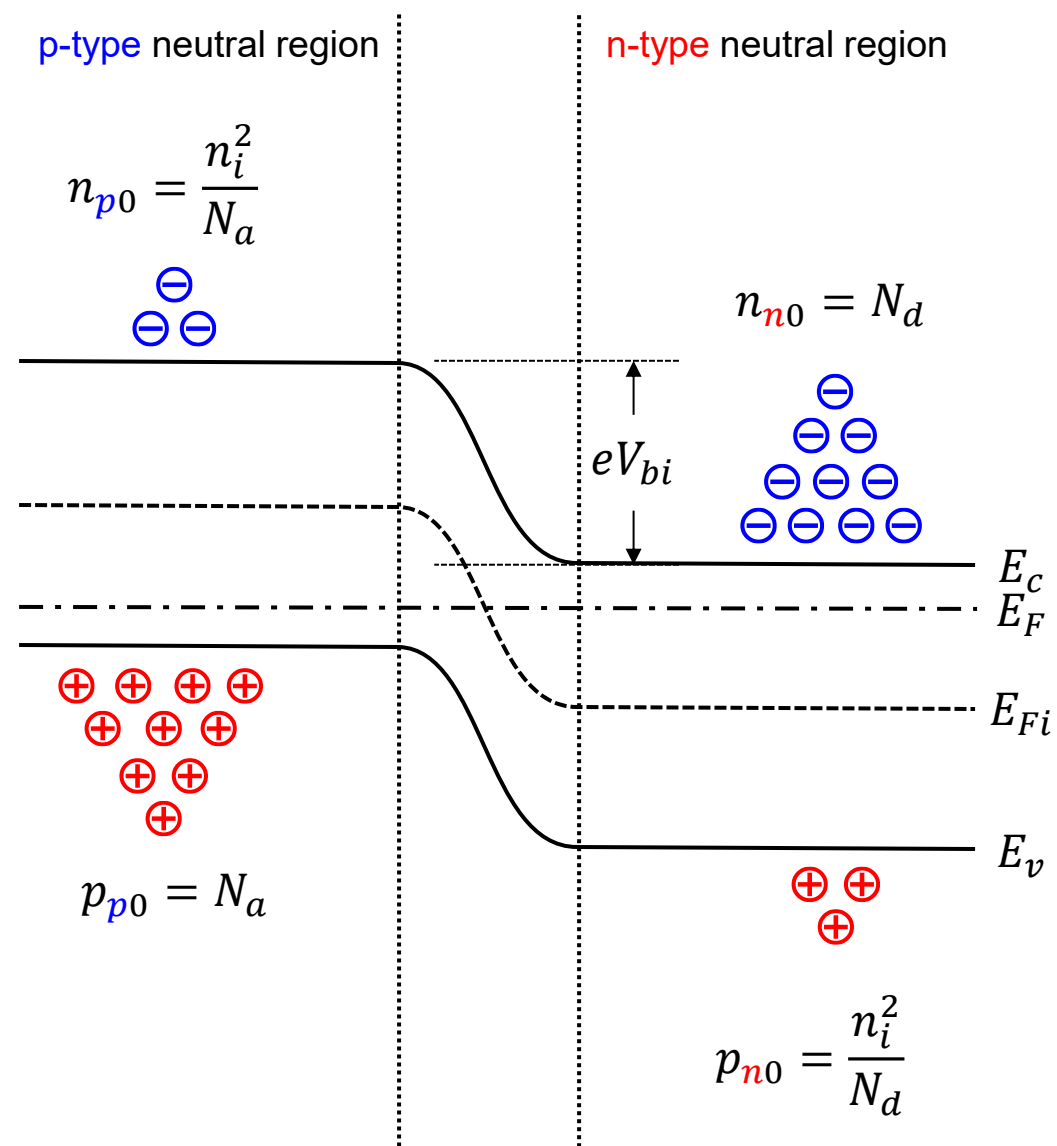
# Terms and Notation for PN Junction

**Table 8.1** | Commonly used terms and notation for this chapter

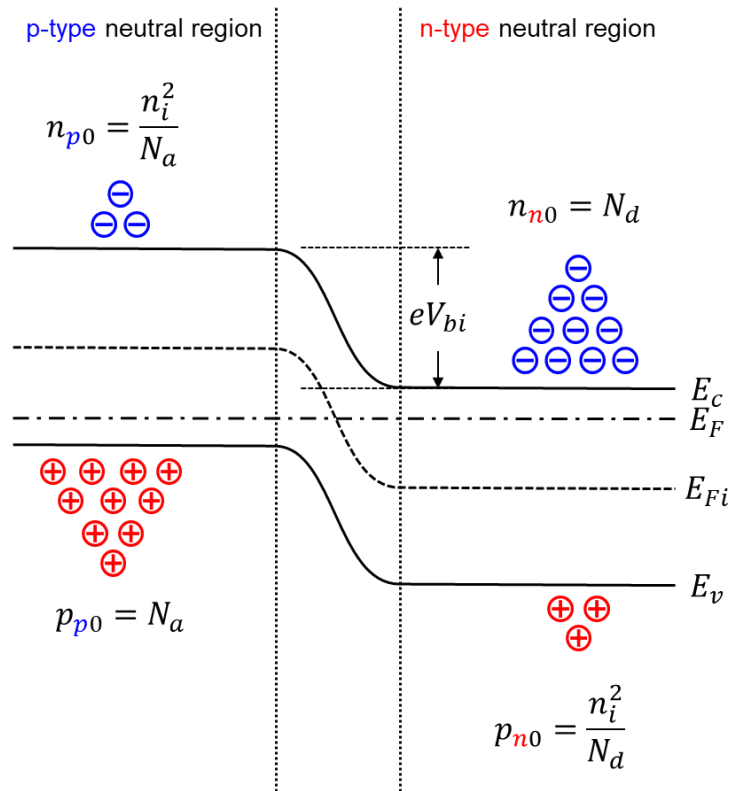
Term	Meaning
$N_a$	Acceptor concentration in the p region of the pn junction
$N_d$	Donor concentration in the n region of the pn junction
$n_{n0} = N_d$	Thermal equilibrium majority carrier electron concentration in the n region
$p_{p0} = N_a$	Thermal equilibrium majority carrier hole concentration in the p region
$n_{p0} = n_i^2 / N_a$	Thermal equilibrium minority carrier electron concentration in the p region
$p_{n0} = n_i^2 / N_d$	Thermal equilibrium minority carrier hole concentration in the n region
$n_p$	Total minority carrier electron concentration in the p region
$p_n$	Total minority carrier hole concentration in the n region
$n_p(-x_p)$	Minority carrier electron concentration in the p region at the space-charge edge
$p_n(x_n)$	Minority carrier hole concentration in the n region at the space charge edge
$\delta n_p = n_p - n_{p0}$	Excess minority carrier electron concentration in the p region
$\delta p_n = p_n - p_{n0}$	Excess minority carrier hole concentration in the n region

# Terms and Notation for PN Junction under Thermal Equilibrium (Zero Bias)

- Assume steady state



# Zero Bias (Steady State)



Built-in potential barrier

$$V_{bi} = \frac{kT}{e} \ln \left( \frac{N_d N_a}{n_i^2} \right) = V_t \ln \left( \frac{N_d N_a}{n_i^2} \right) \quad V_t = \frac{kT}{e}$$



$$\frac{n_i^2}{N_a N_d} = \exp \left( -\frac{eV_{bi}}{kT} \right)$$



$$\frac{n_i^2}{N_a} = N_d \exp \left( -\frac{eV_{bi}}{kT} \right)$$

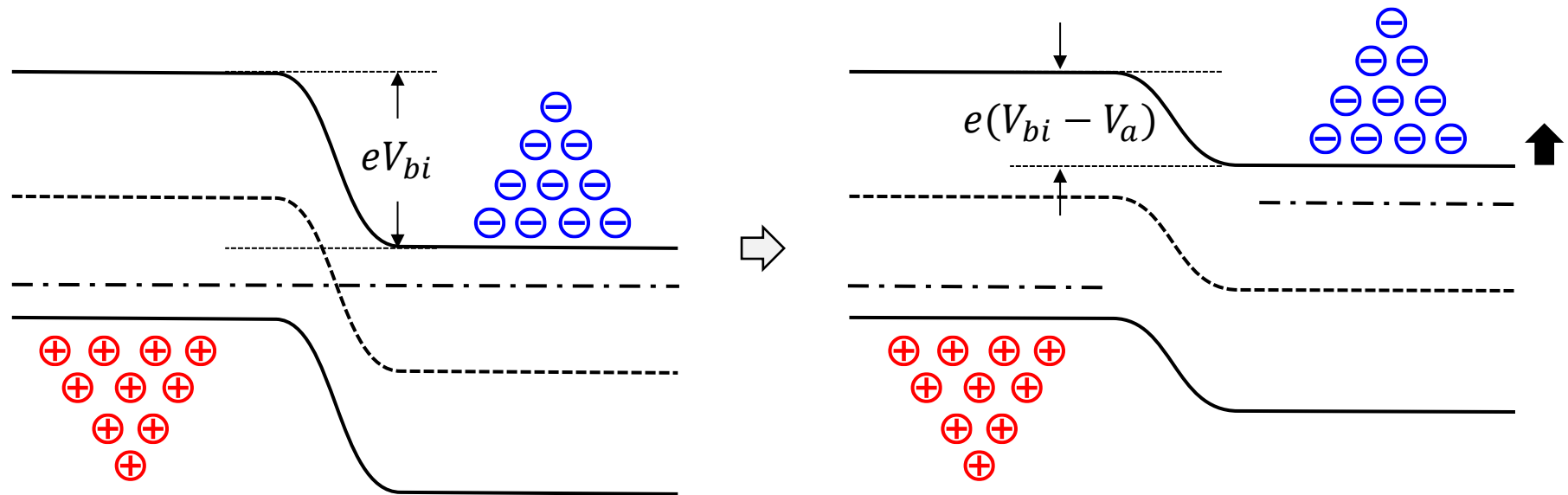


$$n_{p0} = \frac{n_i^2}{N_a} \quad n_{n0} = N_d$$

$$n_{p0} = n_{n0} \exp \left( -\frac{eV_{bi}}{kT} \right) = N_d \exp \left( -\frac{eV_{bi}}{kT} \right)$$

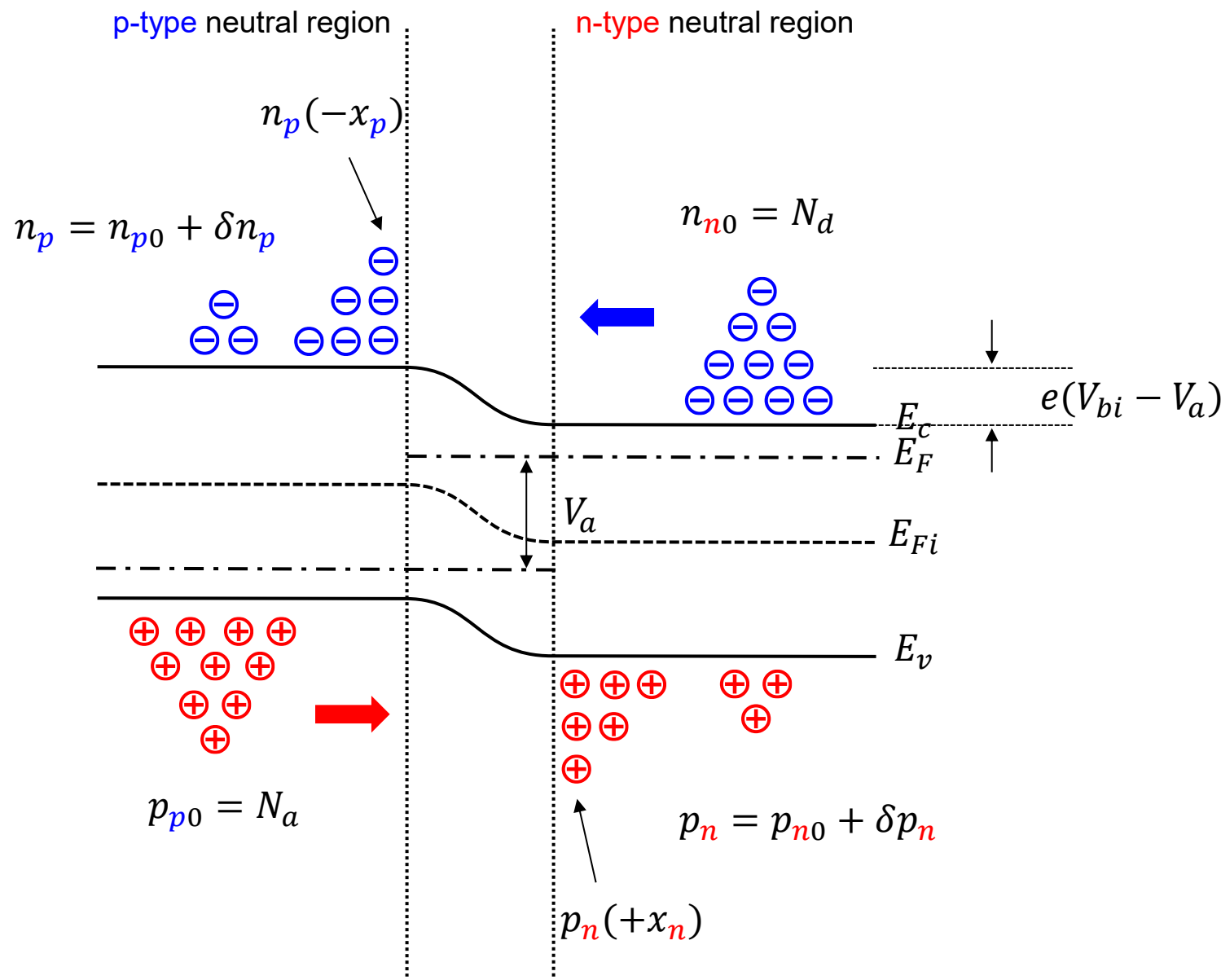
$$p_{n0} = p_{p0} \exp \left( -\frac{eV_{bi}}{kT} \right) = N_a \exp \left( -\frac{eV_{bi}}{kT} \right)$$

# Zero Bias and Forward Bias (Steady State)



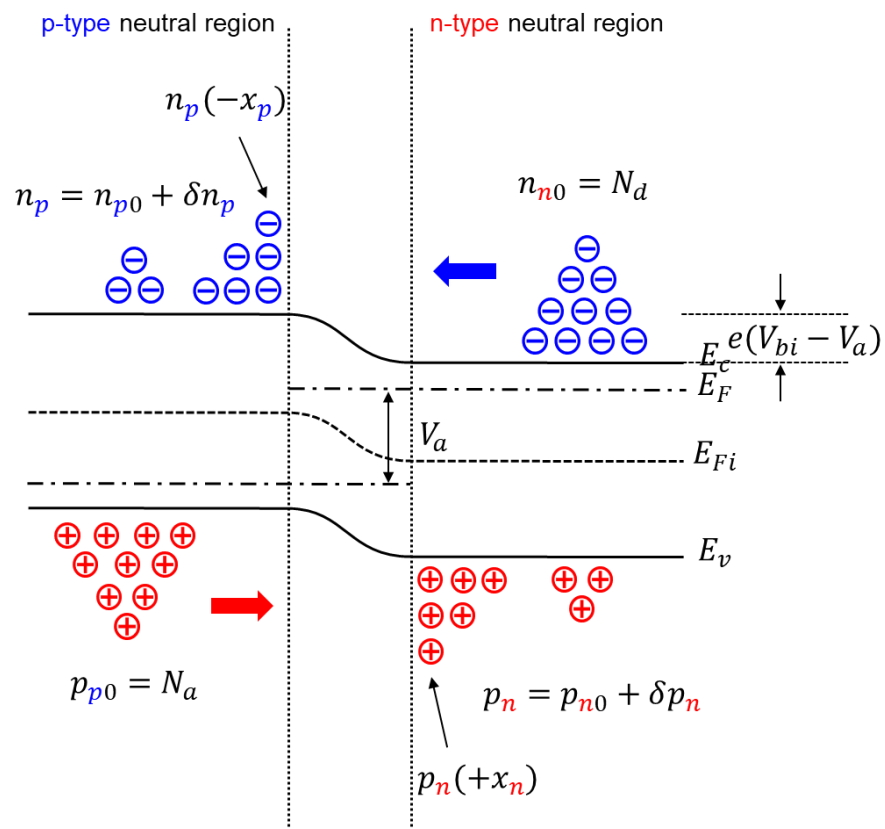
# Terms and Notation for PN Junction under Forward Bias

- Assume steady state





# Forward Bias (Steady State)



For thermal equilibrium

$$n_{p0} = n_{n0} \exp\left(-\frac{eV_{bi}}{kT}\right) = N_d \exp\left(-\frac{eV_{bi}}{kT}\right)$$

Under application of **Forward** bias

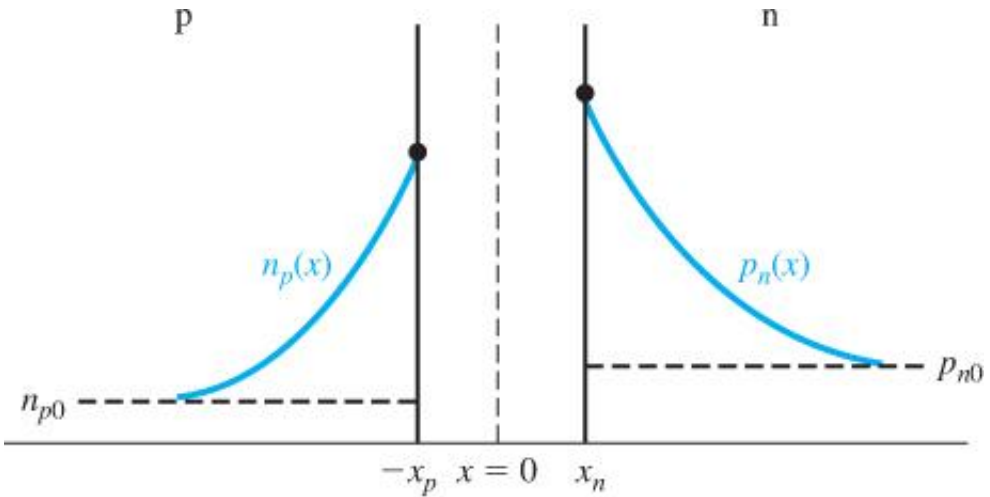
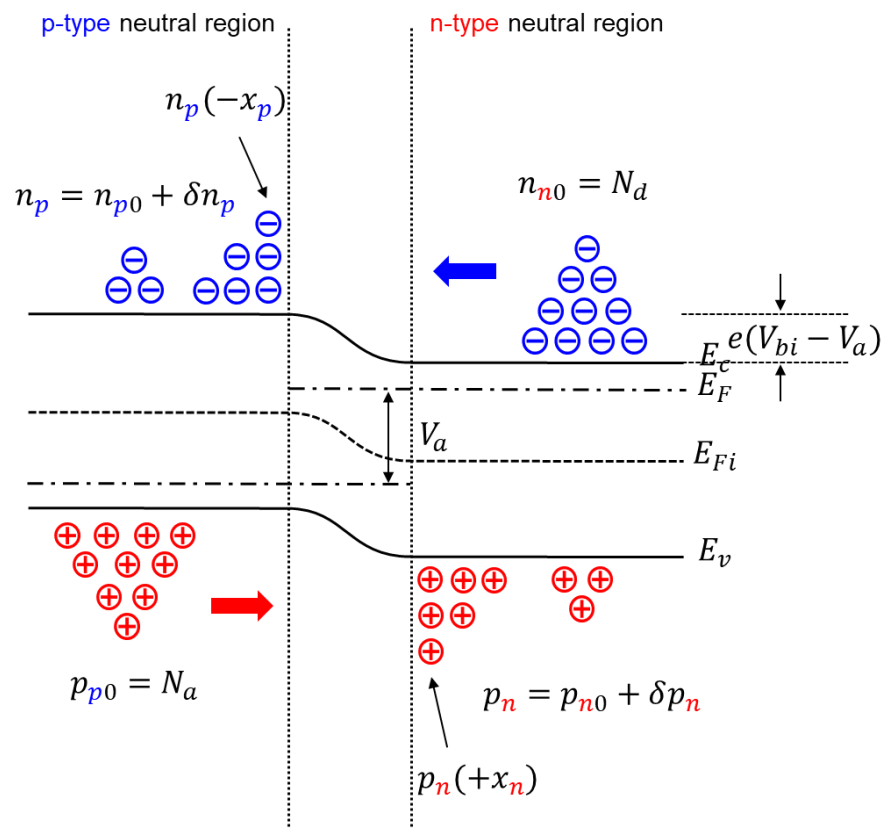
$$\begin{aligned} n_p &= n_{n0} \exp\left(-\frac{e(V_{bi} - V_a)}{kT}\right) \\ &= n_{n0} \exp\left(-\frac{eV_{bi}}{kT}\right) \exp\left(\frac{eV_a}{kT}\right) = n_{p0} \exp\left(\frac{eV_a}{kT}\right) \end{aligned}$$

$$p_n = p_{n0} \exp\left(\frac{eV_a}{kT}\right)$$

$$\begin{aligned} n_p &= n_{p0} \exp\left(\frac{eV_a}{kT}\right) \\ p_n &= p_{n0} \exp\left(\frac{eV_a}{kT}\right) \end{aligned} \Rightarrow$$

$$\begin{aligned} n_p(-x_p) &= n_{p0} \exp\left(\frac{eV_a}{kT}\right) \\ p_n(+x_n) &= p_{n0} \exp\left(\frac{eV_a}{kT}\right) \end{aligned}$$

# Forward Bias (Steady State)



**Figure 8.5** | Steady-state minority carrier concentrations in a pn junction under forward bias.

$$n_p = n_{p0} \exp\left(\frac{eV_a}{kT}\right)$$

$$p_n = p_{n0} \exp\left(\frac{eV_a}{kT}\right)$$

⇒

$$n_p(-x_p) = n_{p0} \exp\left(\frac{eV_a}{kT}\right)$$

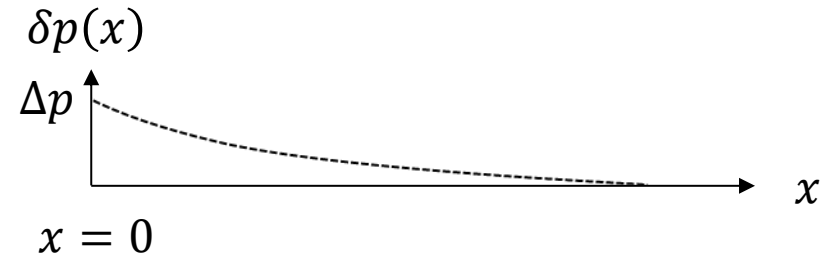
$$p_n(+x_n) = p_{n0} \exp\left(\frac{eV_a}{kT}\right)$$

# Ambipolar Transport (Example)

Additional assumptions

- No excess carrier generation
- No external (applied) E-field
- Steady-state condition

Light illumination



For n-type

$$\frac{d^2 \delta N(x)}{dx^2} = \frac{\delta N(x)}{D_p \tau} \quad L_p = \sqrt{D_p \tau} \quad \Rightarrow \quad \delta p(x) = A e^{+\frac{x}{L}} + B e^{-\frac{x}{L}}$$

Boundary condition

$$x \rightarrow \infty, \quad \delta p(x) \rightarrow 0 \quad \Rightarrow \quad A = 0$$

$$x = 0, \quad \delta p(0) = \Delta p \quad \Rightarrow \quad B = \Delta p$$

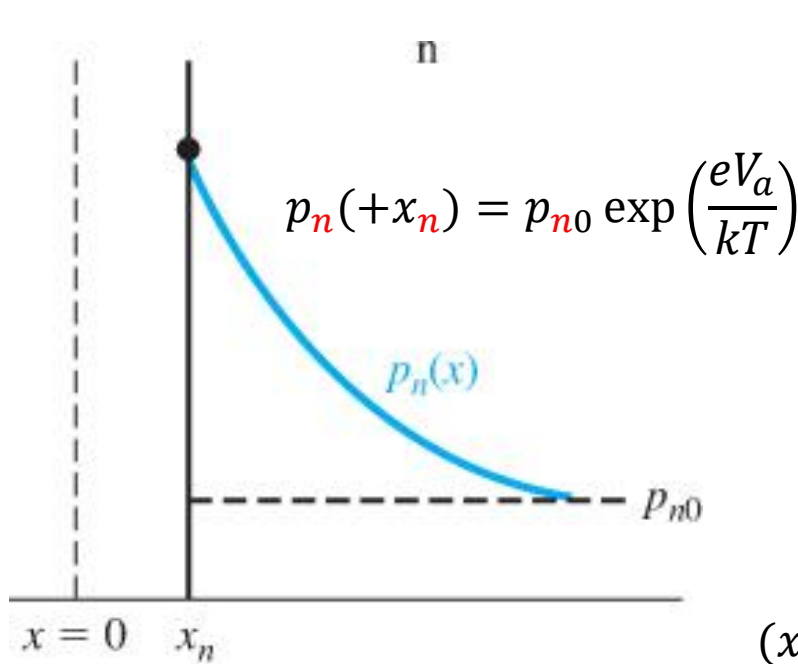
Excess carrier concentration

$$\Rightarrow \quad \delta p(x) = \Delta p e^{-\frac{x}{L_p}}$$

Current density

$$J_p(x) = J_{p|diff} = -q D_p \frac{dp(x)}{dx} = -q D_p \frac{d\delta p(x)}{dx} = \frac{q D_p \Delta p}{L_p} e^{-\frac{x}{L_p}} = q \frac{D_p}{L_p} \delta p(x)$$

# Forward Bias (Steady State) for n-type Region



Boundary conditions

$$x = +x_n \Rightarrow p_n(+x_n) = p_{n0} \exp\left(\frac{eV_a}{kT}\right)$$

$$\begin{aligned} \delta p_n(+x_n) &= p_{n0} \exp\left(\frac{eV_a}{kT}\right) - p_{n0} \\ &= p_{n0} \left[ \exp\left(\frac{eV_a}{kT}\right) - 1 \right] \end{aligned}$$

$$x \rightarrow \infty \Rightarrow p_n(x \rightarrow \infty) = p_{n0}$$

$$\delta p_n(x \rightarrow \infty) = 0$$

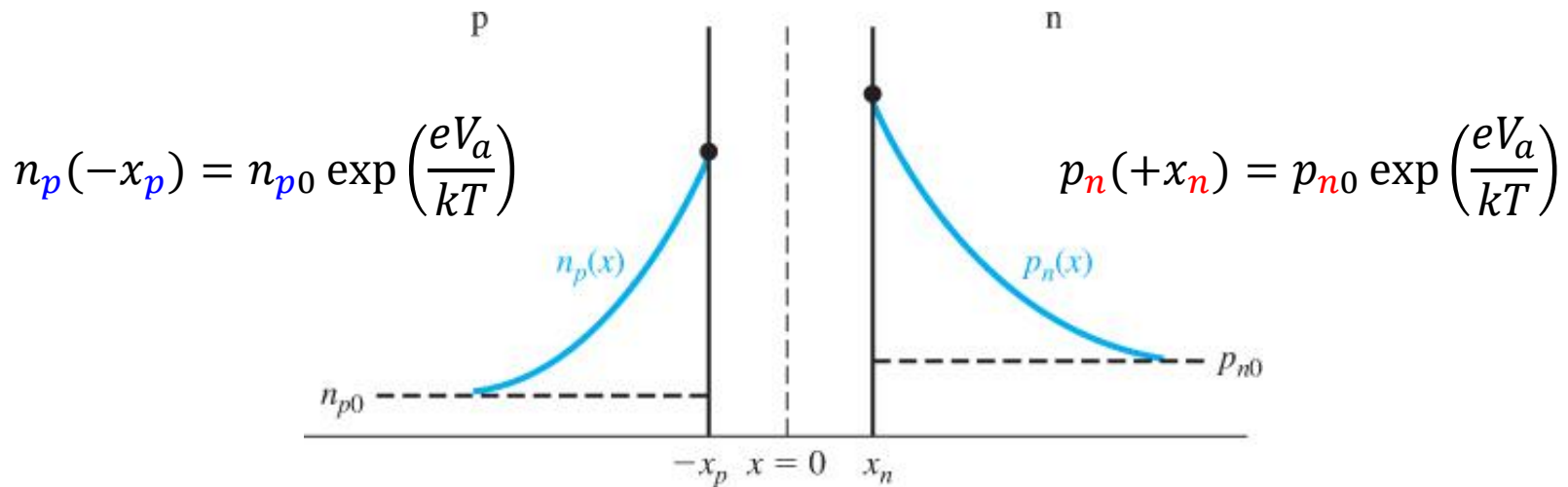
$$\begin{aligned} & (x \geq +x_n) \\ & (x \rightarrow x - x_n) \\ \delta p_n(x) &= A \exp\left(+\frac{x}{L_p}\right) + B \exp\left(-\frac{x}{L_p}\right) \Rightarrow \delta p_n(x) = A \exp\left(+\frac{x - x_n}{L_p}\right) + B \exp\left(-\frac{x - x_n}{L_p}\right) \end{aligned}$$

$$x \rightarrow \infty \Rightarrow \delta p_n(x \rightarrow \infty) = 0, \quad A = 0$$

$$x = +x_n \Rightarrow \delta p_n(+x_n) = B = p_{n0} \left[ \exp\left(\frac{eV_a}{kT}\right) - 1 \right]$$

$$\Rightarrow \delta p_n(x) = p_{n0} \left[ \exp\left(\frac{eV_a}{kT}\right) - 1 \right] \exp\left(-\frac{x - x_n}{L_p}\right) \quad (x \geq +x_n)$$

# Forward Bias (Steady State)



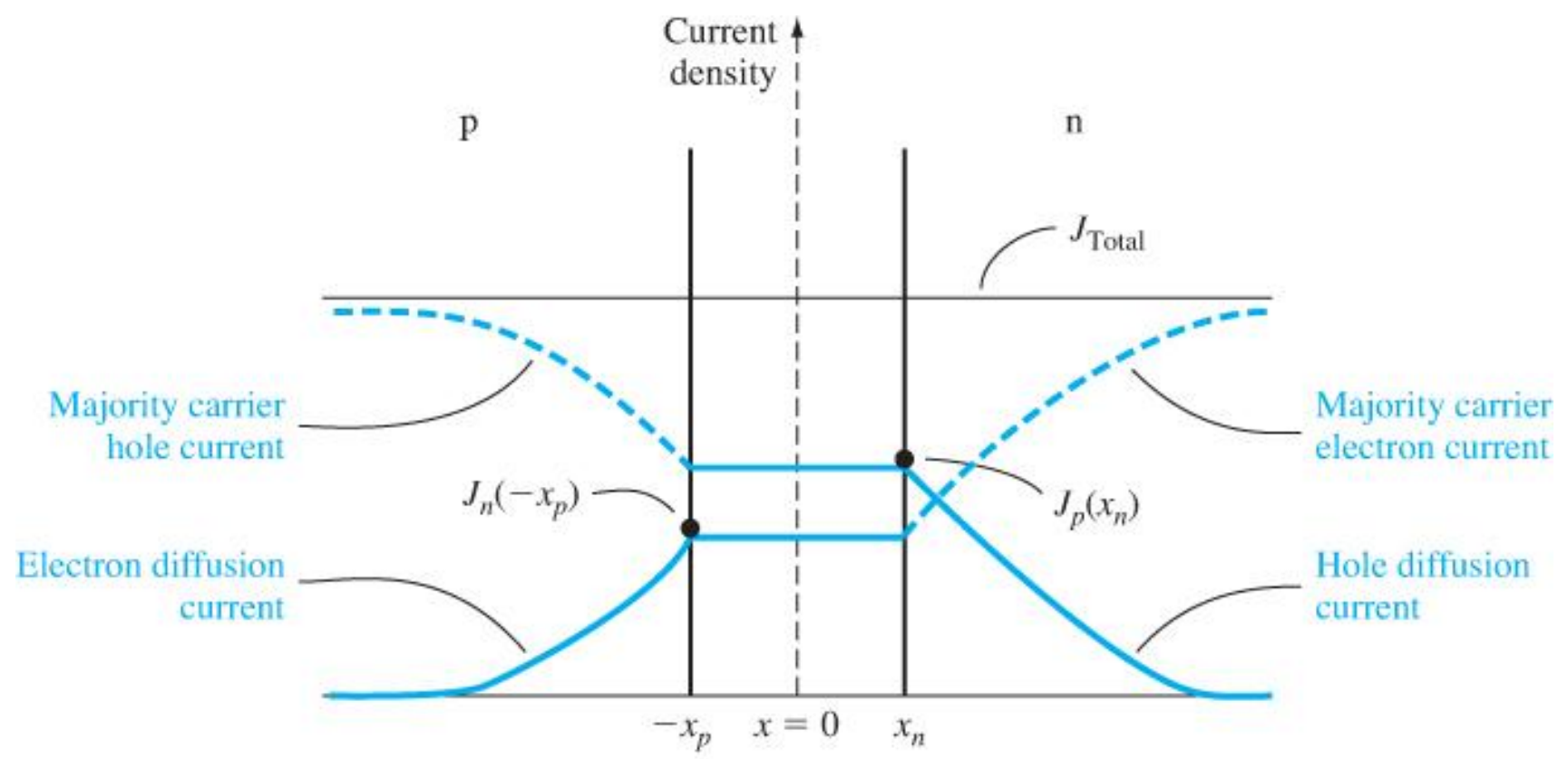
For n-type region

$$\begin{aligned} \delta p_n(x) &= p_{n0} \left[ \exp\left(\frac{eV_a}{kT}\right) - 1 \right] \exp\left(-\frac{x - x_n}{L_p}\right) \\ &= p_{n0} \left[ \exp\left(\frac{eV_a}{kT}\right) - 1 \right] \exp\left(\frac{x_n - x}{L_p}\right) \end{aligned} \quad (x \geq +x_n)$$

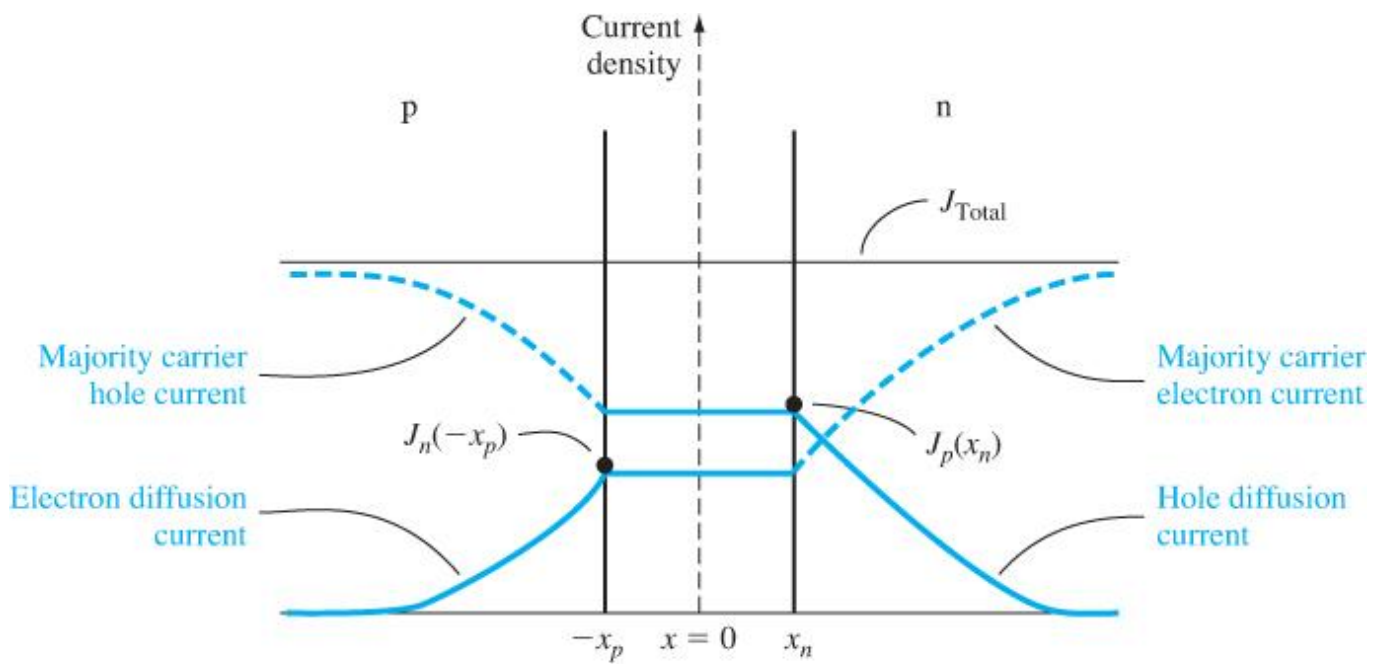
For p-type region

$$\begin{aligned} \delta n_p(x) &= n_{p0} \left[ \exp\left(\frac{eV_a}{kT}\right) - 1 \right] \exp\left(-\frac{-(x + x_p)}{L_n}\right) \\ &= n_{p0} \left[ \exp\left(\frac{eV_a}{kT}\right) - 1 \right] \exp\left(\frac{x_p + x}{L_n}\right) \end{aligned} \quad (x \leq -x_p)$$

# Ideal PN Junction Current under Forward Bias



# Ideal PN Junction Current under Forward Bias



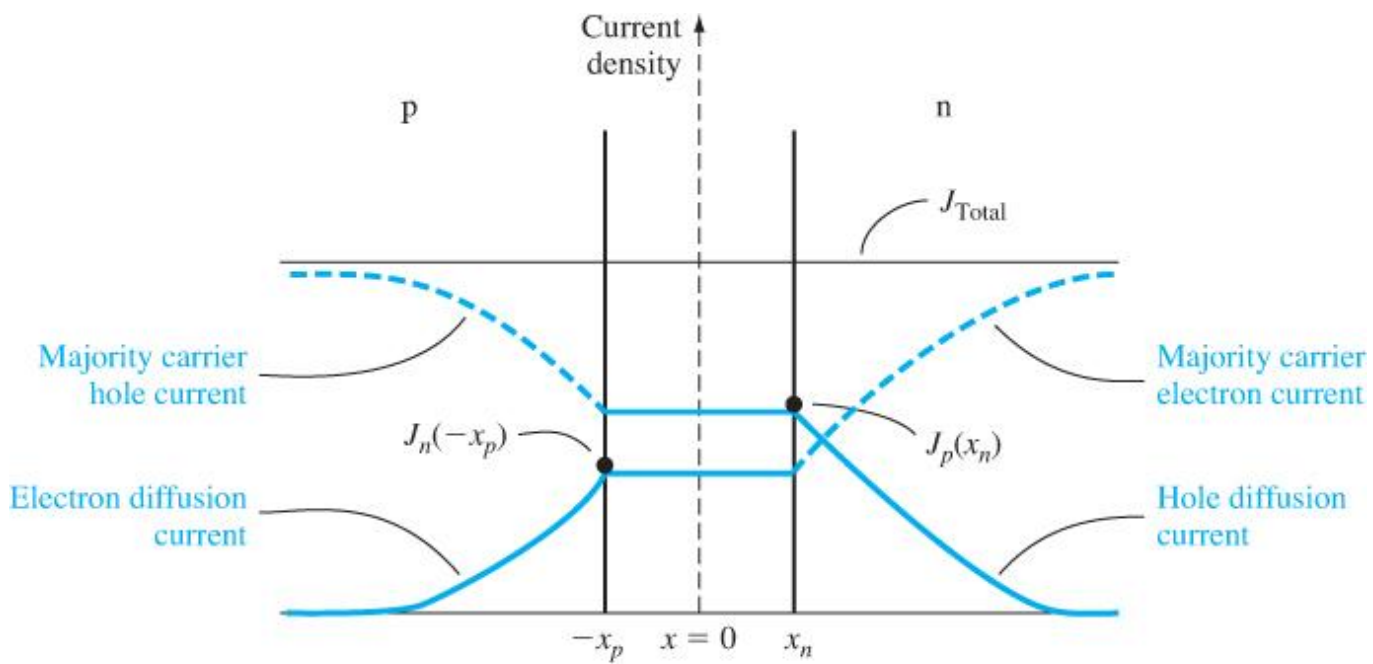
For n-type region

$$J_p(x) = J_{p|diff} = -qD_p \frac{dp_n(x)}{dx} = -eD_p \frac{d\delta p_n(x)}{dx}$$

$$\Downarrow \quad \delta p_n(x) = p_{n0} \left[ \exp\left(\frac{eV_a}{kT}\right) - 1 \right] \exp\left(\frac{x_n - x}{L_p}\right)$$

$$J_p(x) = \frac{eD_p}{L_p} p_{n0} \left[ \exp\left(\frac{eV_a}{kT}\right) - 1 \right] \exp\left(\frac{x_n - x}{L_p}\right) \Rightarrow J_p(x_n) = \frac{eD_p}{L_p} p_{n0} \left[ \exp\left(\frac{eV_a}{kT}\right) - 1 \right] \\ (x = +x_n)$$

# Ideal PN Junction Current under Forward Bias



For p-type region

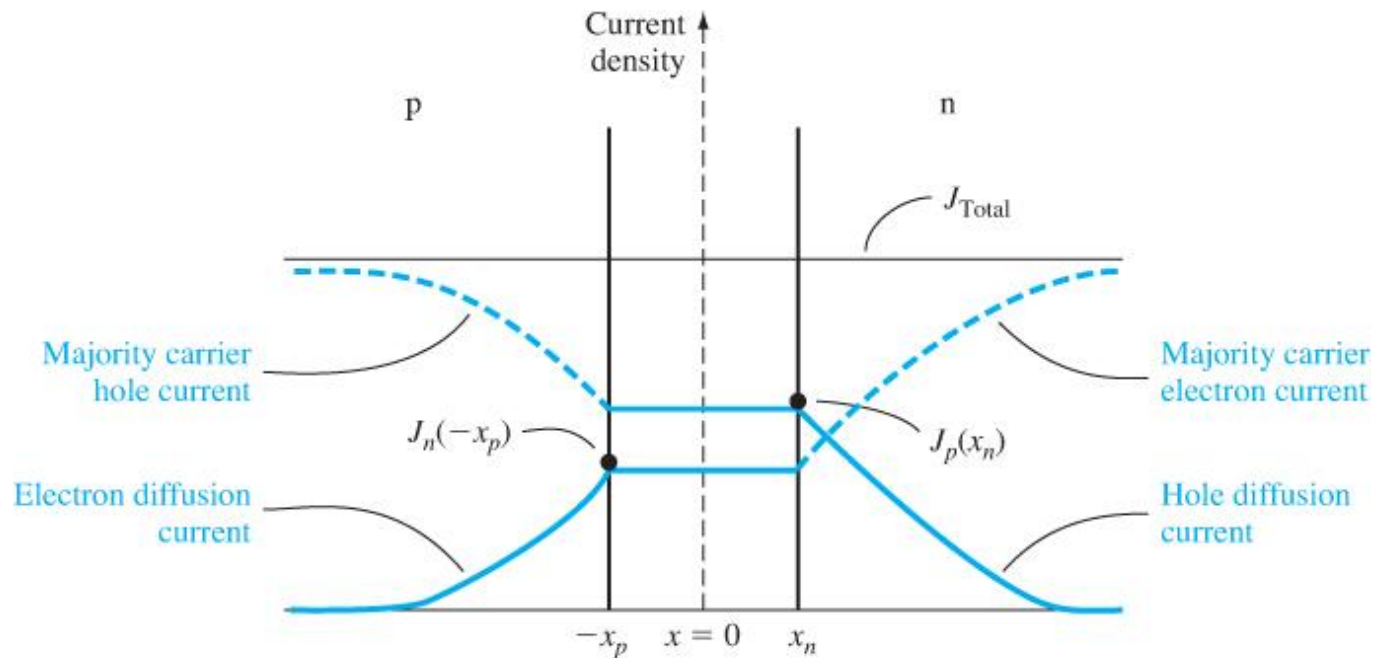
$$J_n(x) = J_{n|diff} = +qD_n \frac{dn_p(x)}{dx} = +eD_n \frac{d\delta n_p(x)}{dx}$$

$$\Downarrow \quad \delta n_p(x) = n_{p0} \left[ \exp\left(\frac{eV_a}{kT}\right) - 1 \right] \exp\left(\frac{x_p + x}{L_n}\right)$$

$$J_n(x) = \frac{eD_n}{L_n} n_{p0} \left[ \exp\left(\frac{eV_a}{kT}\right) - 1 \right] \exp\left(\frac{x_p + x}{L_n}\right) \Rightarrow J_n(-x_p) = \frac{eD_n}{L_n} n_{p0} \left[ \exp\left(\frac{eV_a}{kT}\right) - 1 \right] \quad (x = -x_p)$$



# Ideal PN Junction Current under Forward Bias

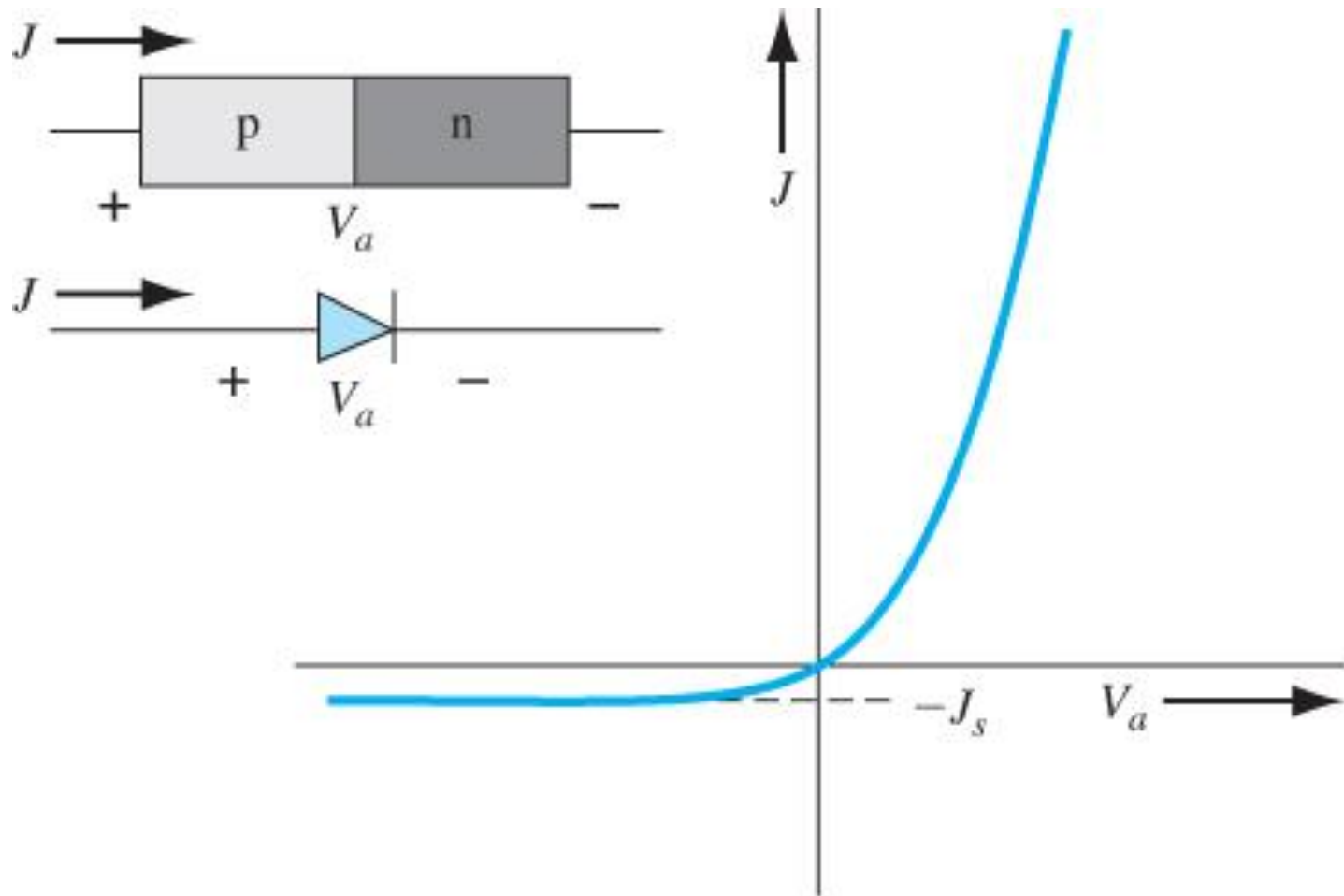


$$J_n(-x_p) = \frac{eD_n}{L_n} n_{p0} \left[ \exp\left(\frac{eV_a}{kT}\right) - 1 \right] \quad J_p(x_n) = \frac{eD_p}{L_p} p_{n0} \left[ \exp\left(\frac{eV_a}{kT}\right) - 1 \right]$$

Depletion region total current density = Total current density

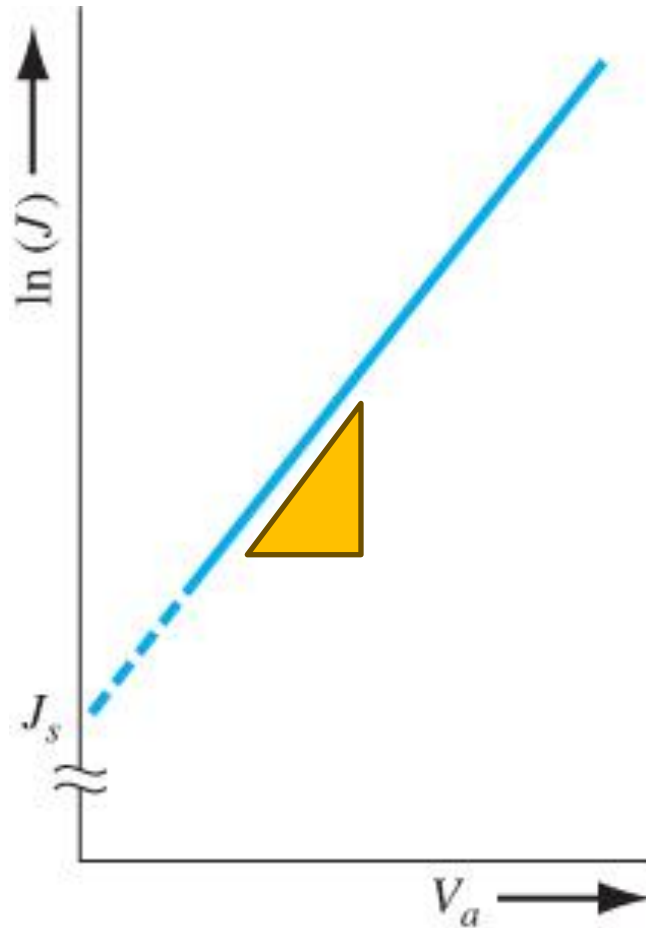
$$\begin{aligned} J_{Total} &= J_p(x_n) + J_n(-x_p) = \left[ \frac{eD_n}{L_n} n_{p0} + \frac{eD_p}{L_p} p_{n0} \right] \left[ \exp\left(\frac{eV_a}{kT}\right) - 1 \right] \\ &= J_s \left[ \exp\left(\frac{eV_a}{kT}\right) - 1 \right] \end{aligned}$$

# Ideal I-V (Current-Voltage) Characteristics of a PN Junction Diode (Linear Plot)



$$J_{Total} = J_s \left[ \exp \left( \frac{eV_a}{kT} \right) - 1 \right]$$

# Ideal I-V (Current-Voltage) Characteristics of a PN Junction Diode (Semilog Plot)



$$\text{if } \exp\left(\frac{eV_a}{kT}\right) \gg 1$$

$$J_{Total} = J_s \left[ \exp\left(\frac{eV_a}{kT}\right) - 1 \right] \cong J_s \exp\left(\frac{eV_a}{kT}\right)$$



$$\begin{aligned} \log(J_{Total}) &\cong \log\left(\exp\left(\frac{eV_a}{kT}\right)\right) + \log J_s \\ &= \frac{\ln\left(\exp\left(\frac{eV_a}{kT}\right)\right)}{\ln 10} + \log J_s = \frac{eV_a}{kT} \frac{1}{\ln 10} + \log J_s \end{aligned}$$



$$\text{Slope} = \frac{d \log(J_{Total})}{dV_a} = \frac{e}{kT} \frac{1}{\ln 10} \cong 16.76$$

$$\frac{1}{\text{Slope}} = \frac{dV_a}{d \log(J_{Total})} \cong \frac{1}{16.76} = 0.060[\text{V/dec}] = 60[\text{mV/dec}]$$

# Temperature Effects

Depletion region total current density = Total current density

$$J_{Total} = J_p(x_{\text{red}}) + J_n(-x_{\text{blue}}) = \left[ \frac{eD_n}{L_n} n_{\text{blue}0} + \frac{eD_p}{L_p} p_{\text{red}0} \right] \left[ \exp\left(\frac{eV_a}{kT}\right) - 1 \right]$$

$$= J_s \left[ \exp\left(\frac{eV_a}{kT}\right) - 1 \right]$$

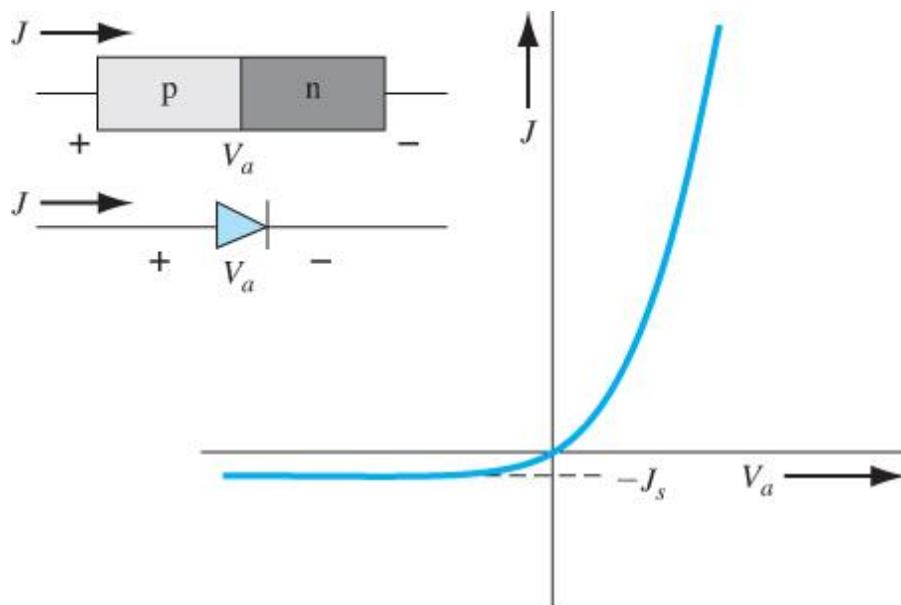
For reverse bias

$$J_{Total} \cong \left[ \frac{eD_n}{L_n} n_{\text{blue}0} + \frac{eD_p}{L_p} p_{\text{red}0} \right] \cong J_s$$

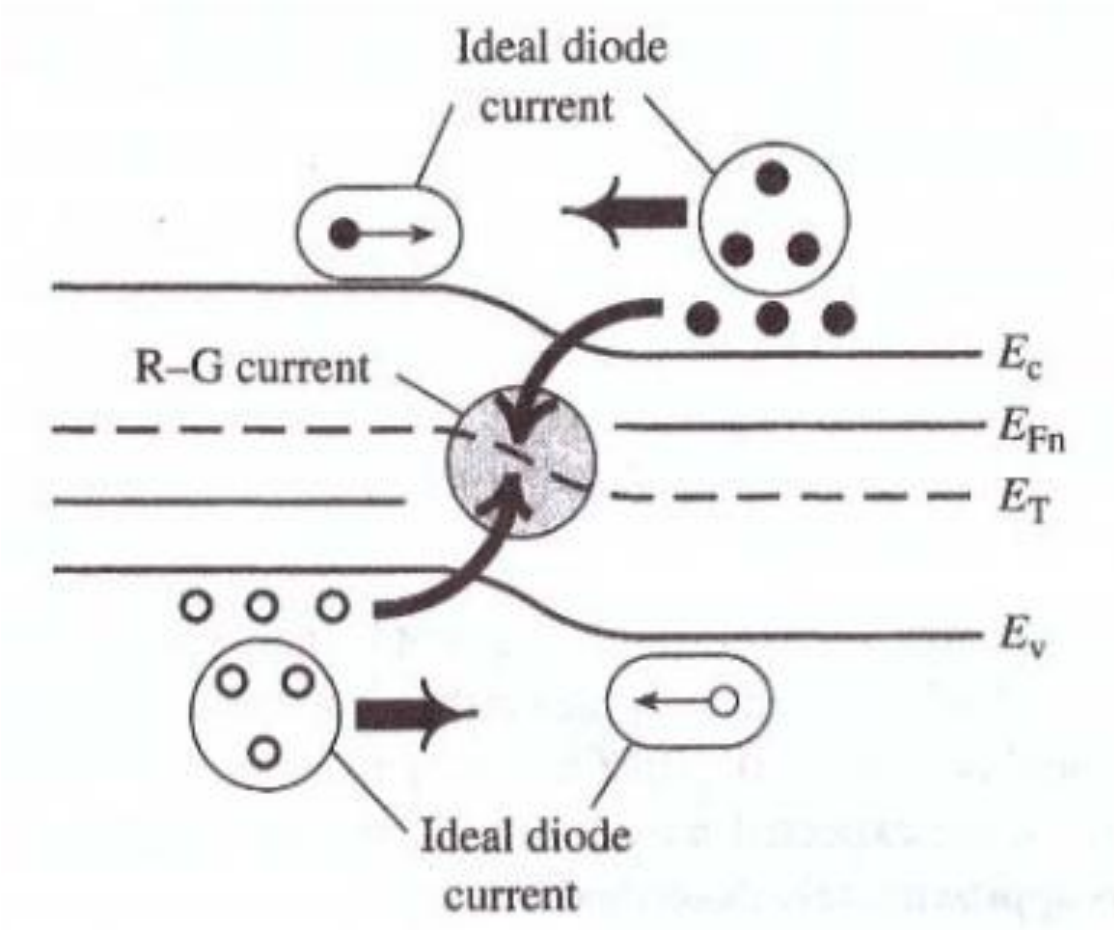
$$n_{\text{blue}0} = \frac{n_i^2}{N_a} \quad p_{\text{red}0} = \frac{n_i^2}{N_d}$$

For forward bias

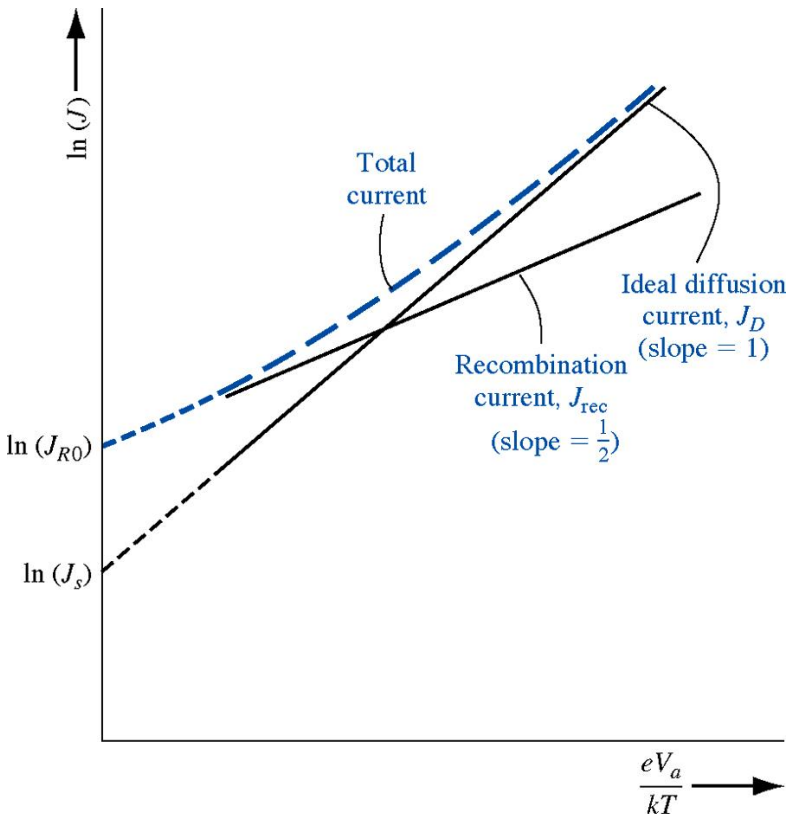
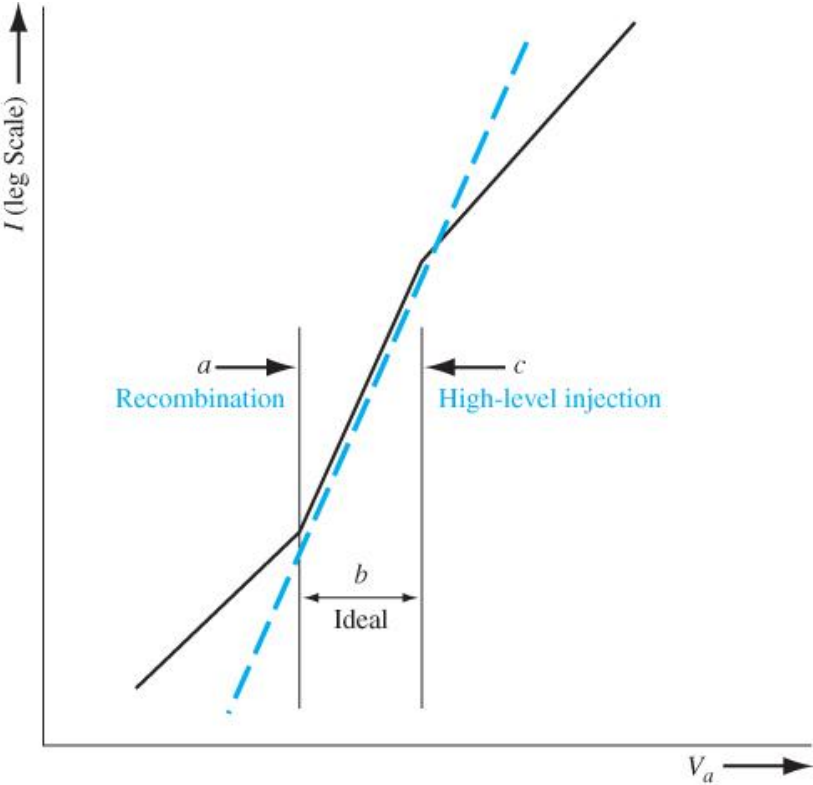
$$J_{Total} \cong J_s \exp\left(\frac{eV_a}{kT}\right)$$



Generation-Recombination Currents (Recombination effects, Forward Bias)



# Generation-Recombination Currents (Recombination effects)



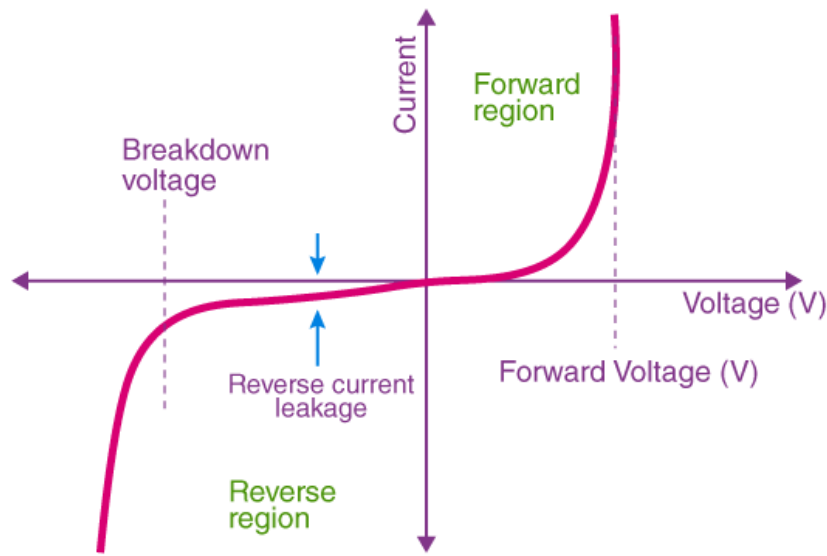
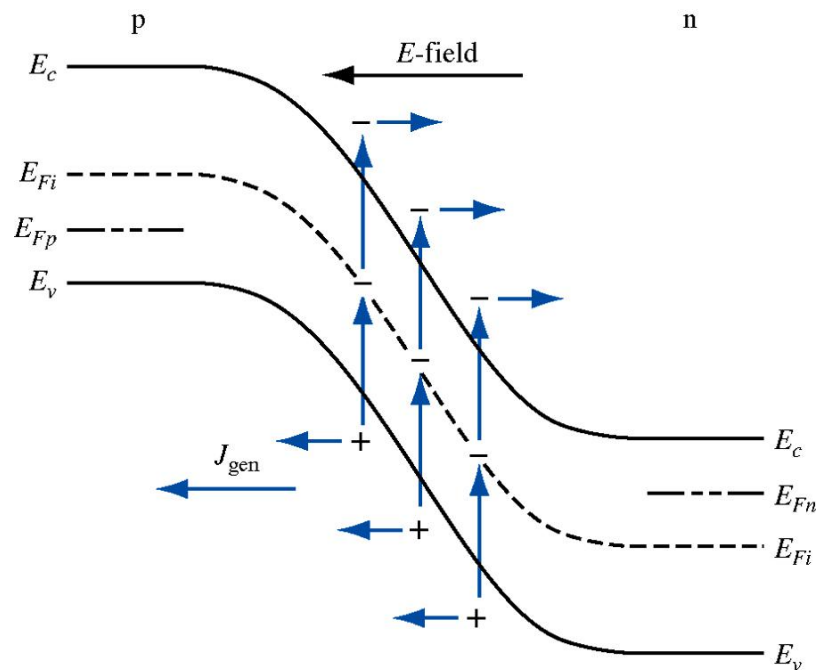
At low bias region with recombination

$$J_{Total} = J_s \exp\left(\frac{eV_a}{kT}\right)$$

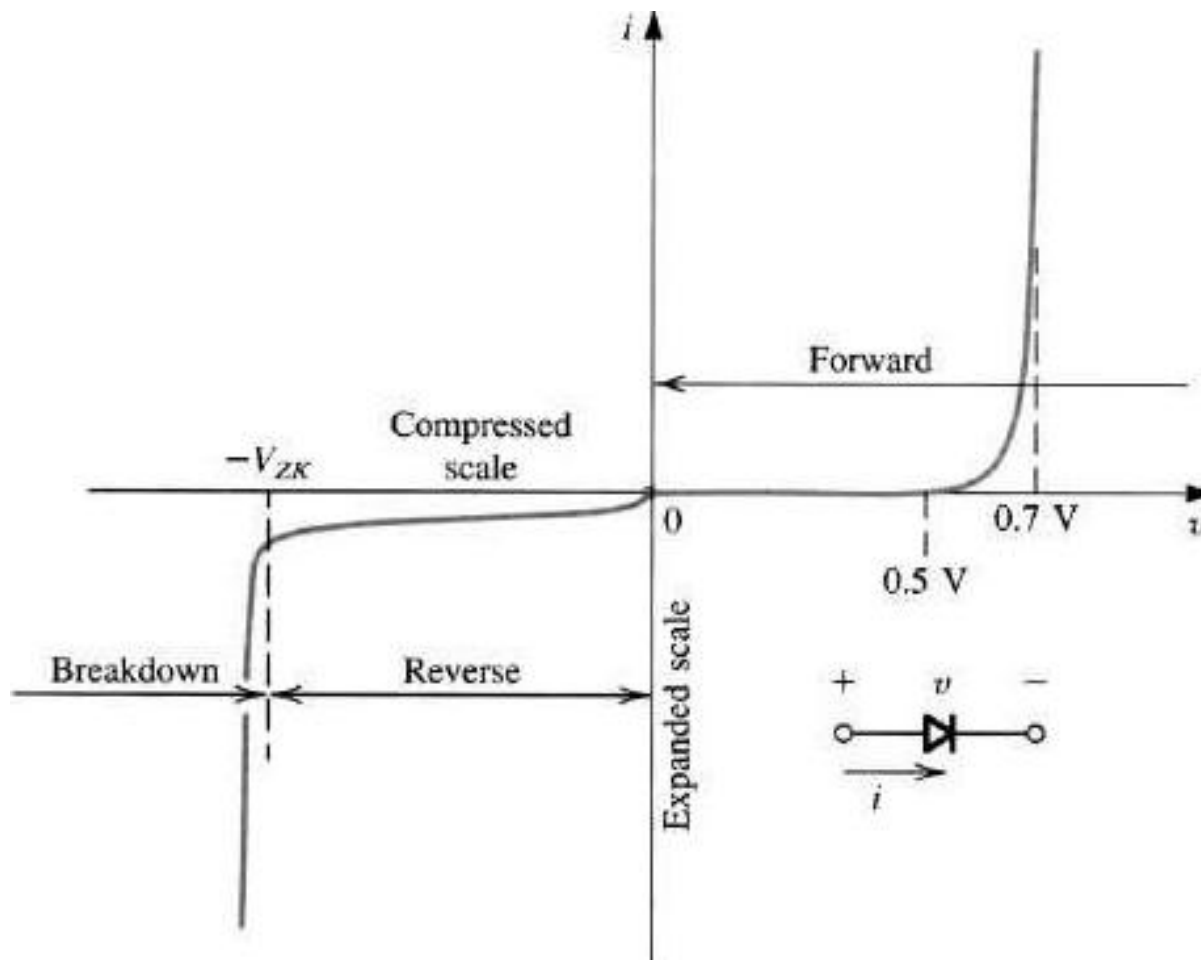
$\Rightarrow$

$$J_{Total} = J_s \exp\left(\frac{eV_a}{2kT}\right)$$

# Generation-Recombination Currents (Generation effects, Reverse Bias)



# I-V Characteristics of a PN Junction Diode Summary





# Thank you



# Q & A

Artificial Intelligence Semiconductor device and Sensor  
(AISS) Lab



Seyong Oh

Assistant Professor  
Division of Electrical Engineering  
Hanyang University ERICA



Website

- ✓ E-mail: [seyongoh89@hanyang.ac.kr](mailto:seyongoh89@hanyang.ac.kr)
- ✓ Website: <https://seyongoh89site.wixsite.com/profseyongoh/about-9>