



Semiconductor Devices

# Chapter 7

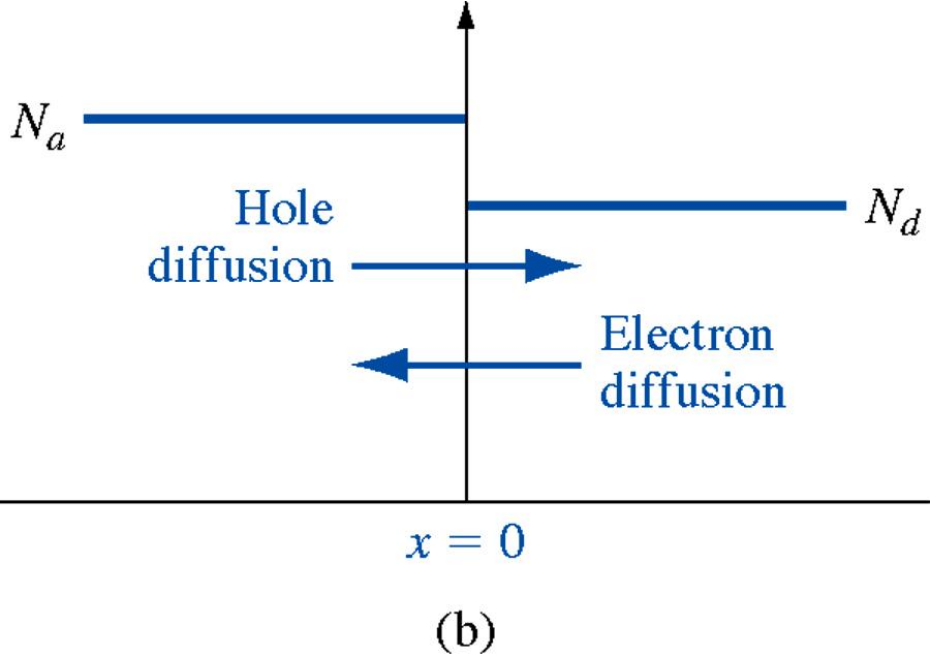
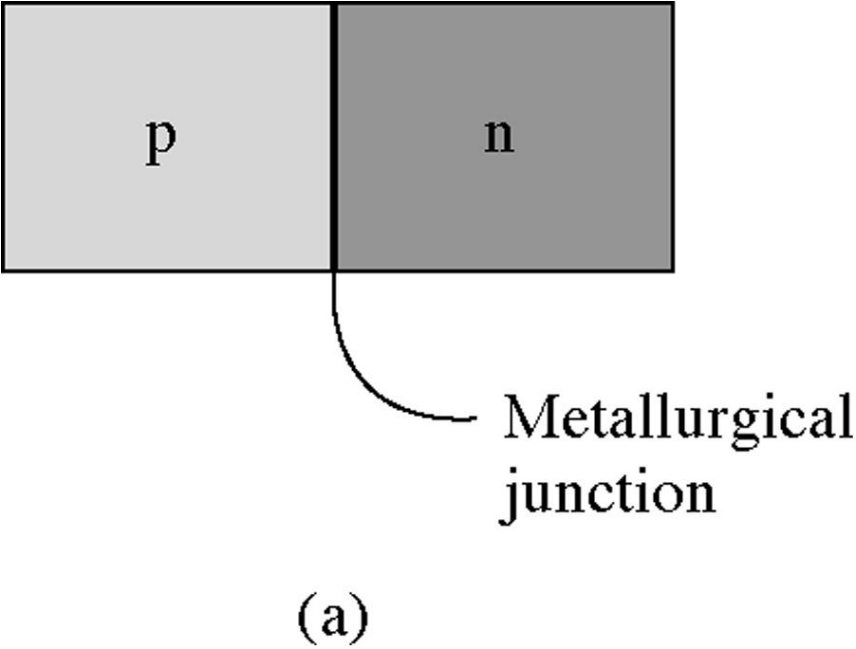
## The pn Junction

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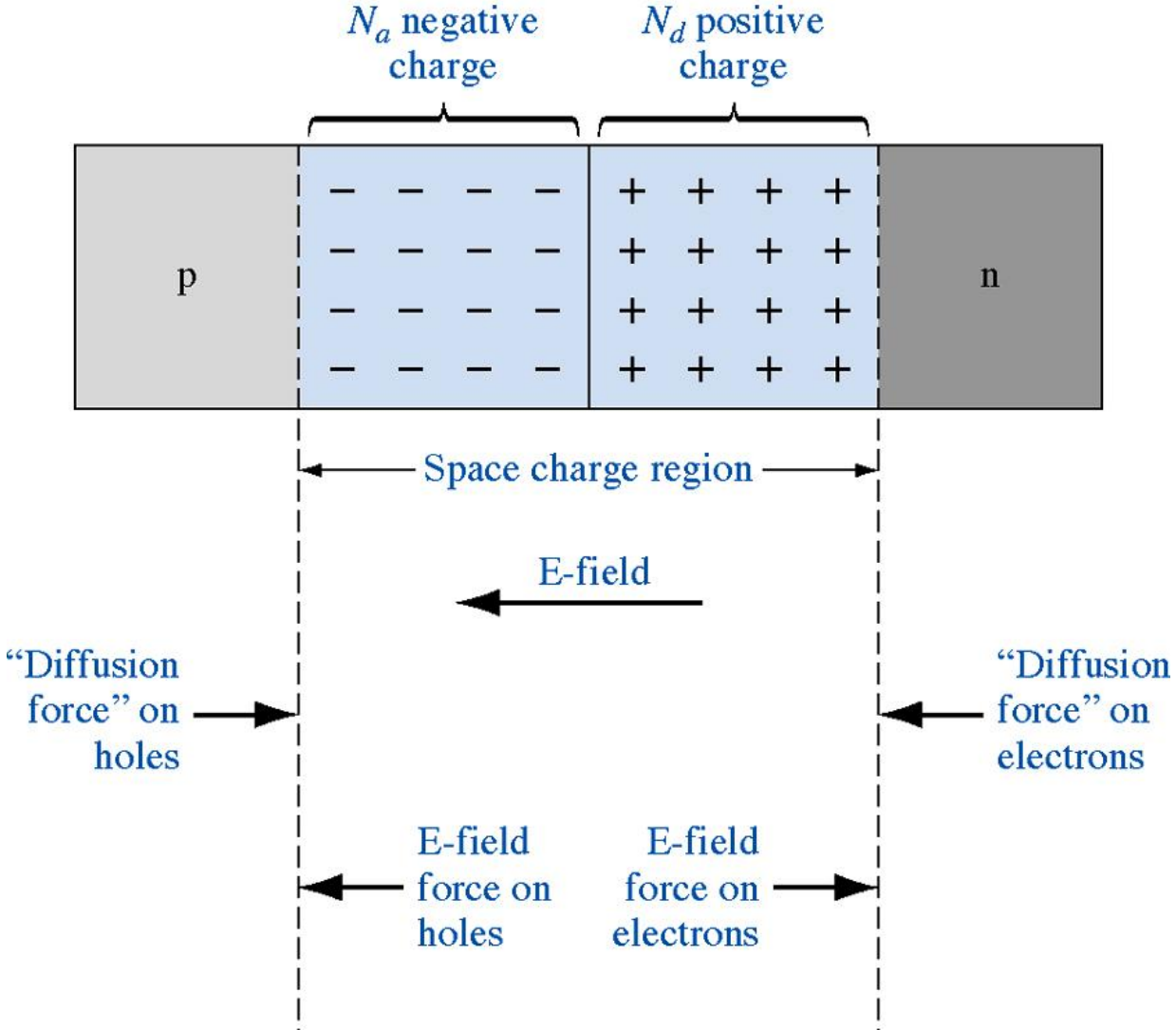


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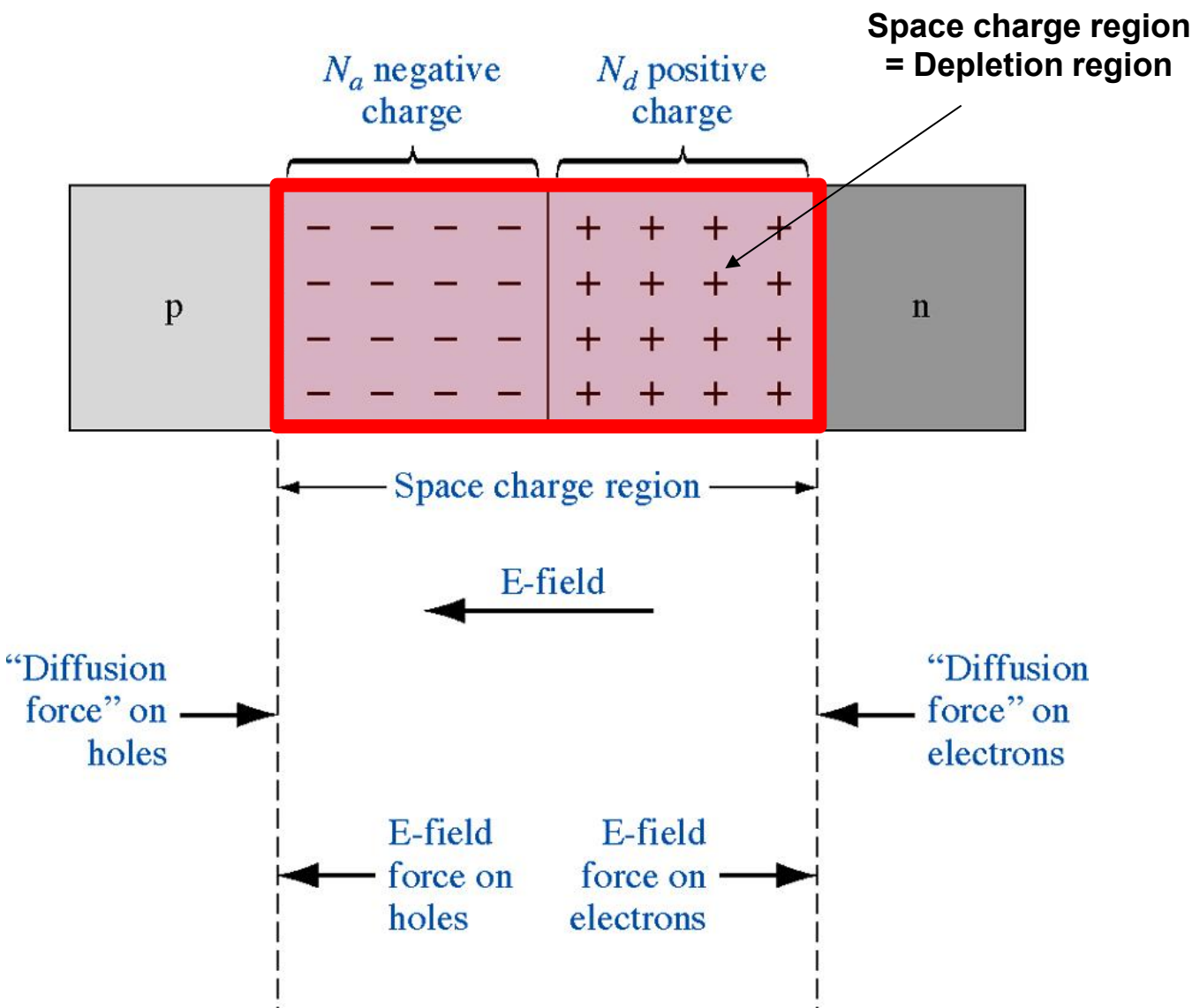
# Basic Structure of PN Junction



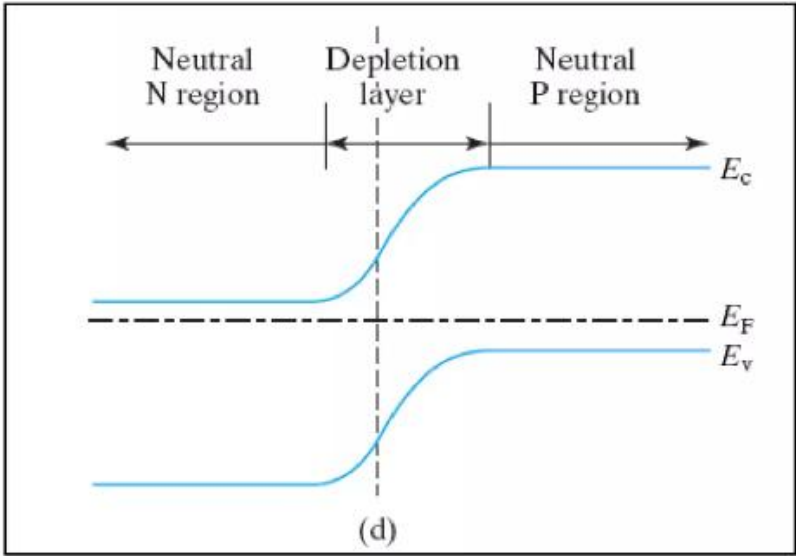
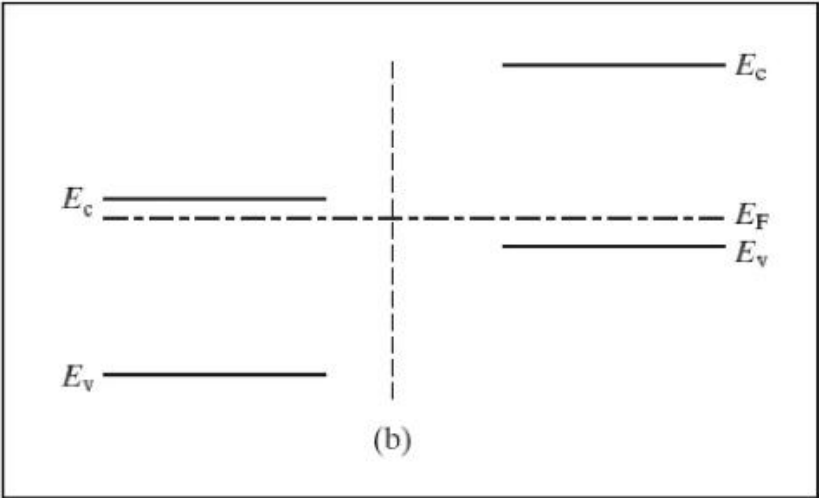
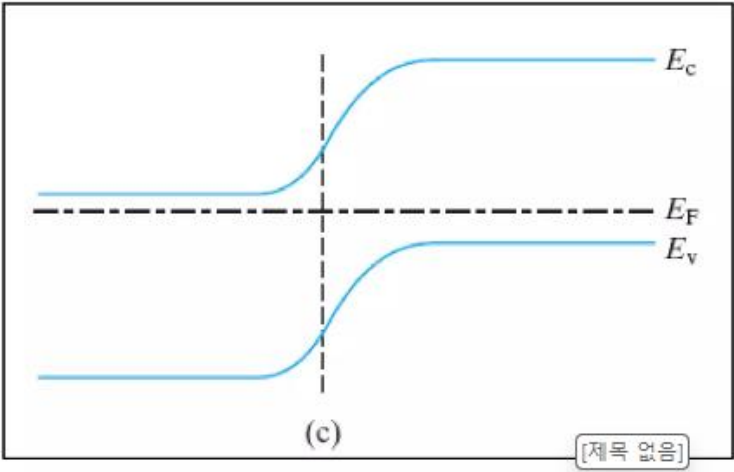
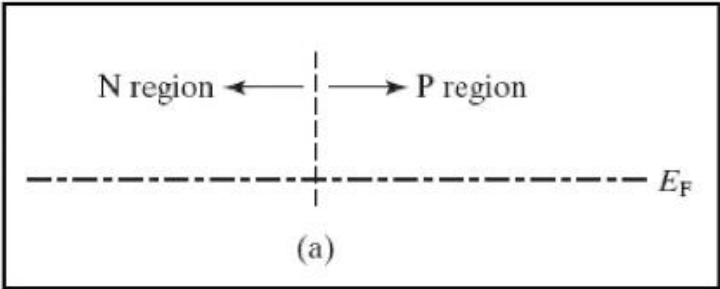
# Basic Structure of PN Junction



# Basic Structure of PN Junction

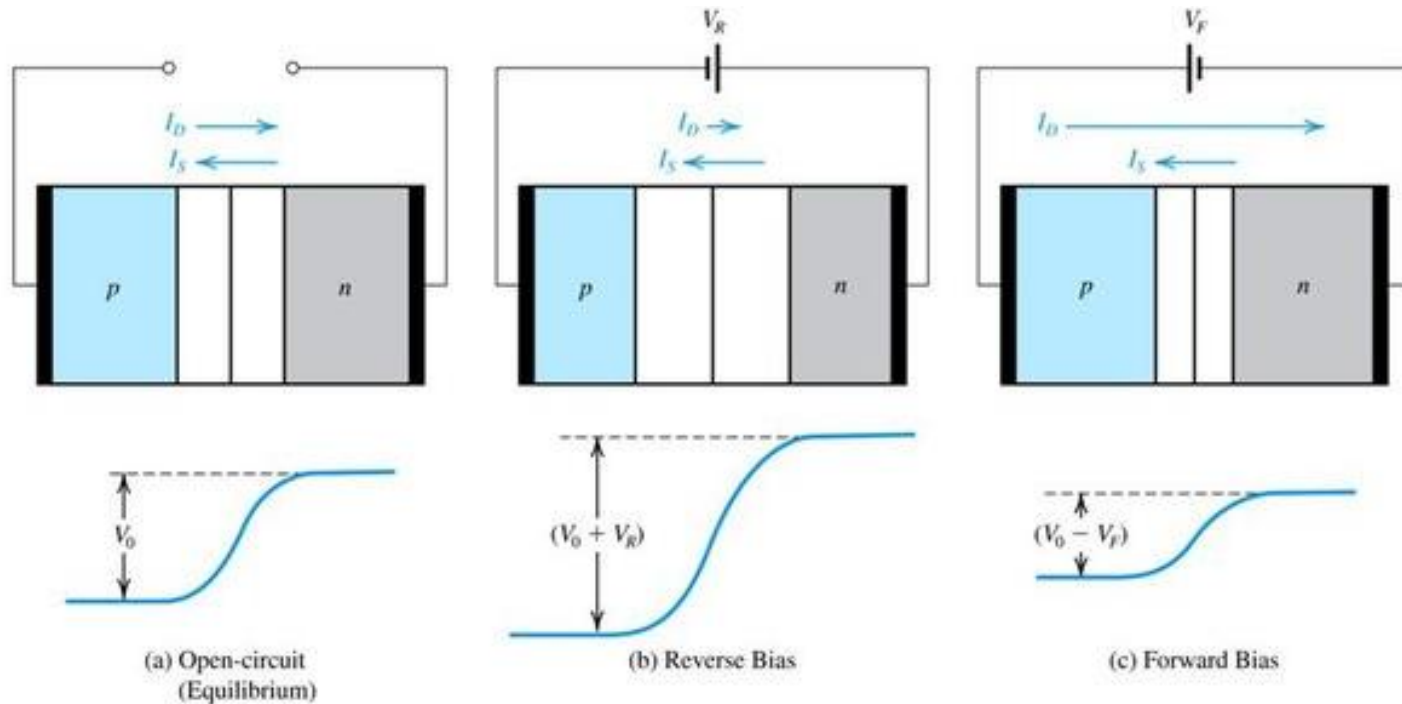


# PN Junction Band Diagram

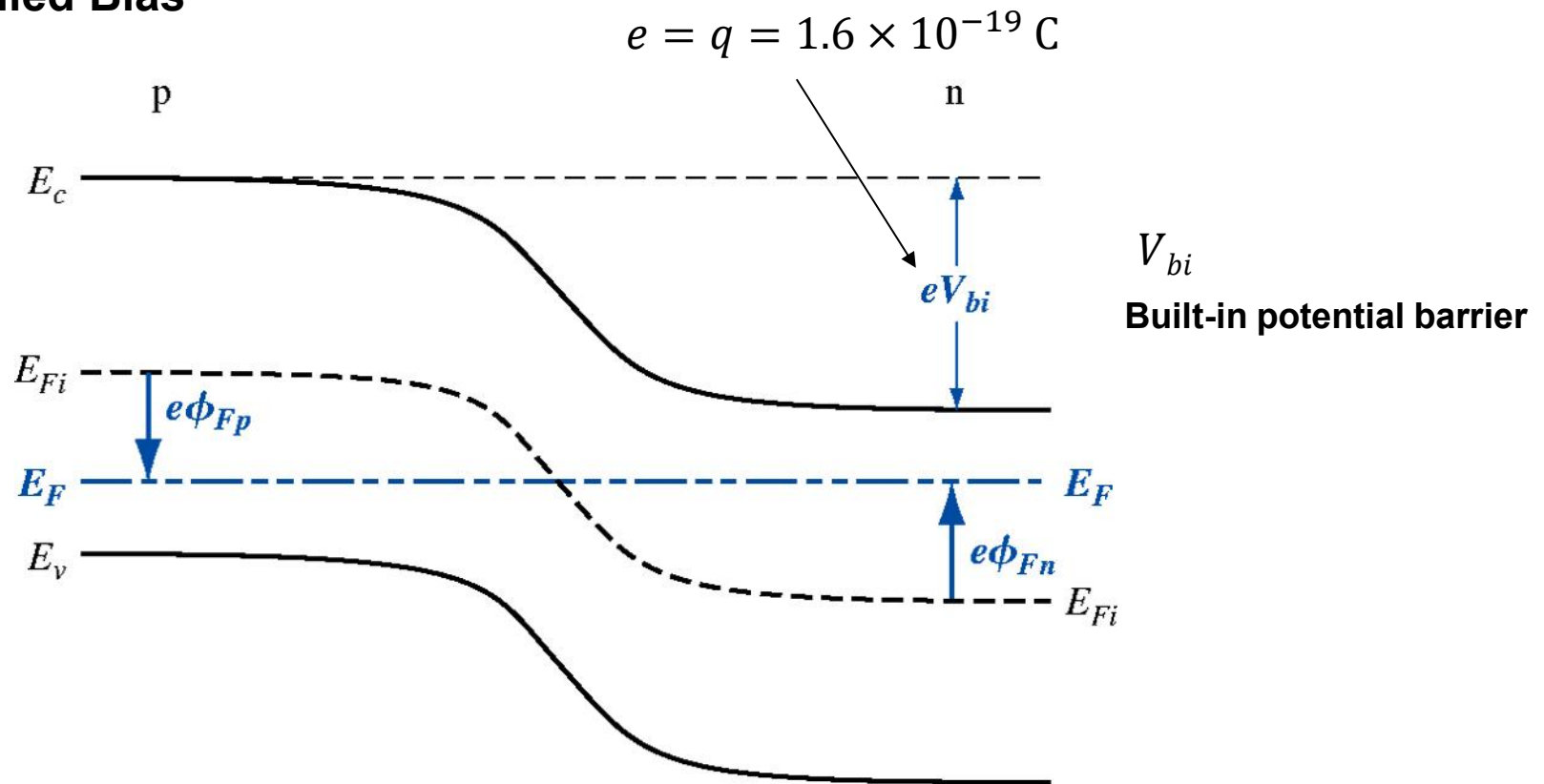


# Voltage Bias

- Zero Bias (Equilibrium)
- Reverse Bias
- Forward Bias



# Zero Applied Bias



$$p_0 = N_a = n_i \exp\left(\frac{E_{Fi} - E_F}{kT}\right)$$

$$= n_i \exp\left(\frac{e\phi_{Fp}}{kT}\right)$$

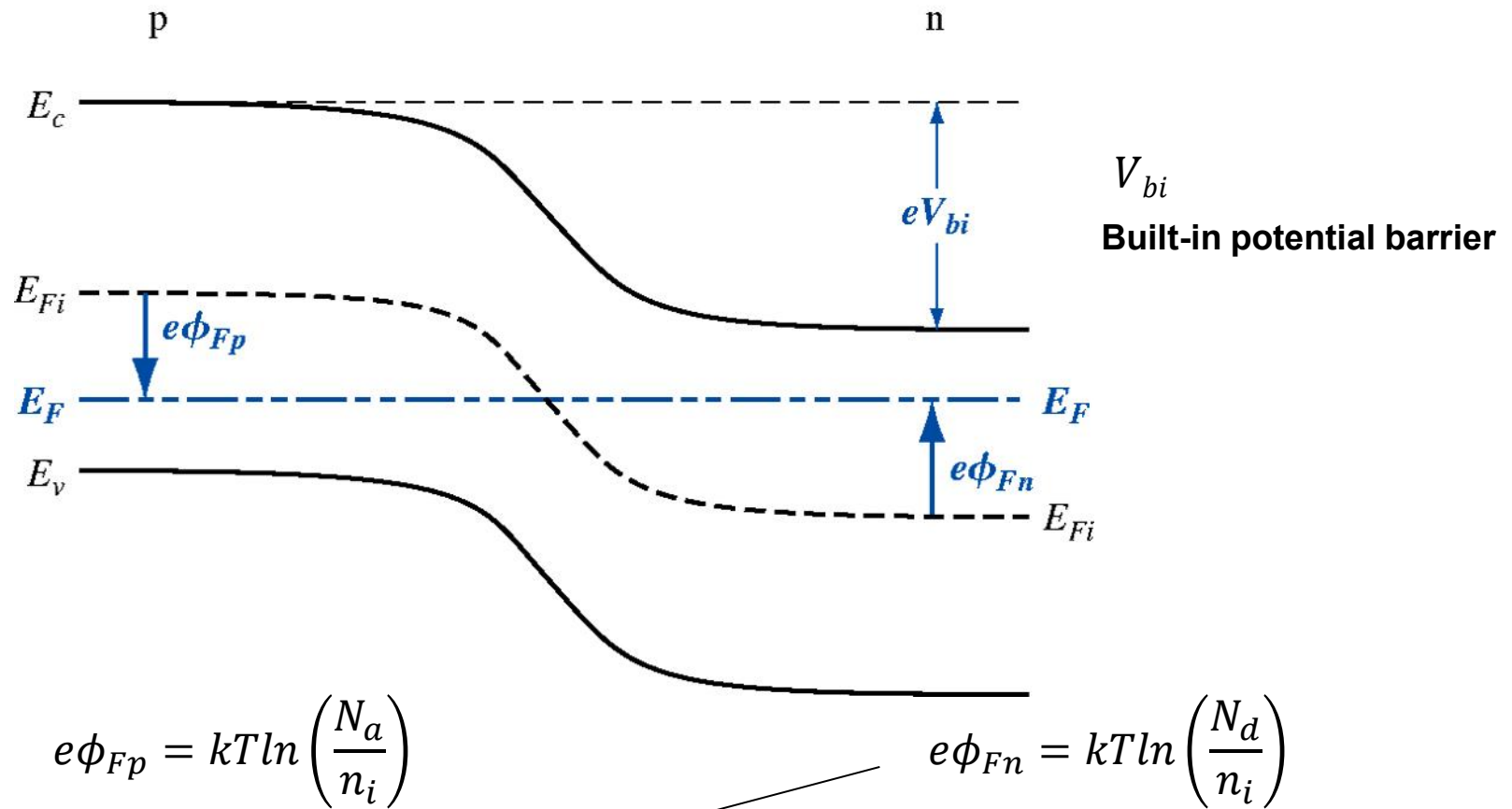
$$e\phi_{Fp} = kT \ln\left(\frac{N_a}{n_i}\right)$$

$$n_0 = N_d = n_i \exp\left(\frac{E_F - E_{Fi}}{kT}\right)$$

$$= n_i \exp\left(\frac{e\phi_{Fn}}{kT}\right)$$

$$e\phi_{Fn} = kT \ln\left(\frac{N_d}{n_i}\right)$$

# Zero Applied Bias



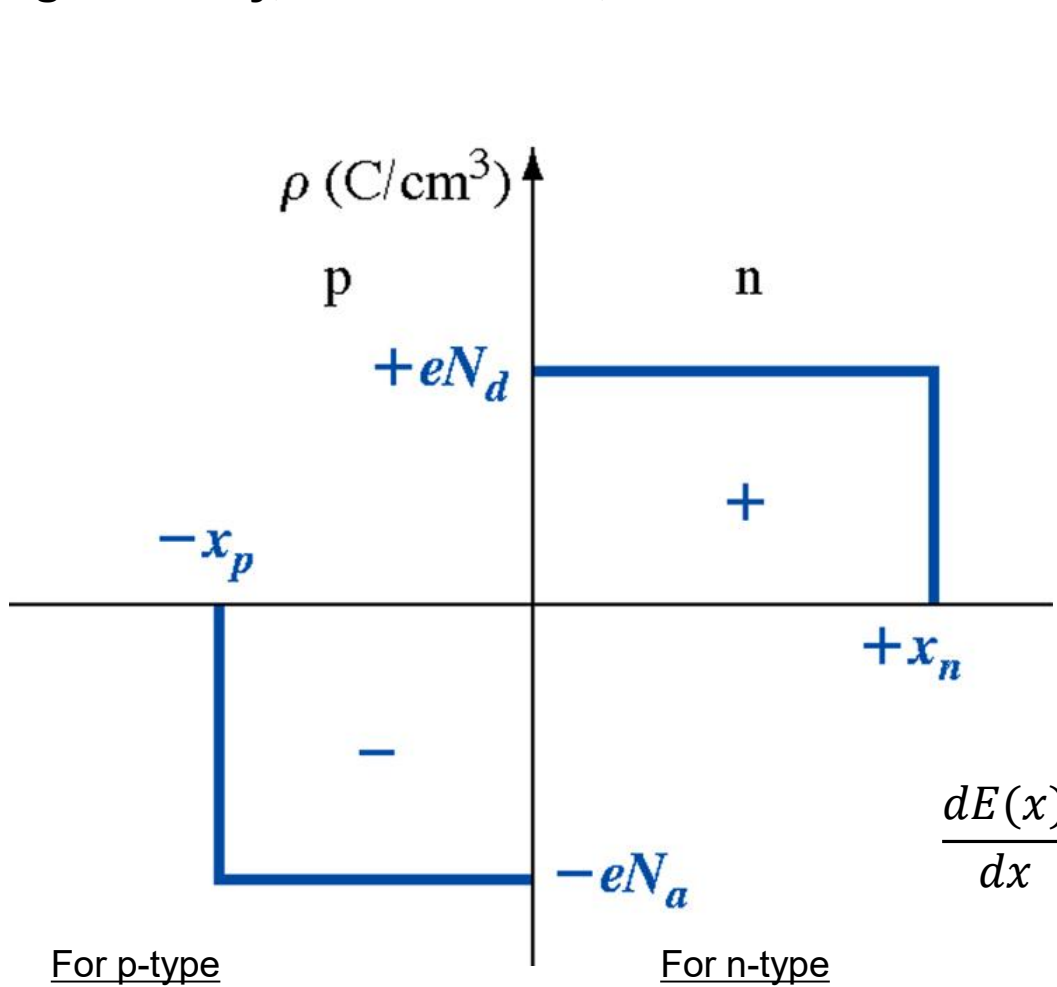
$$eV_{bi} = e\phi_{Fp} + e\phi_{Fn} = kT \ln \left( \frac{N_a}{n_i} \right) + kT \ln \left( \frac{N_d}{n_i} \right) = kT \ln \left( \frac{N_d N_a}{n_i^2} \right)$$

**Built-in potential barrier**

$$V_{bi} = \frac{kT}{e} \ln \left( \frac{N_d N_a}{n_i^2} \right) = V_t \ln \left( \frac{N_d N_a}{n_i^2} \right) \quad V_t = \frac{kT}{e}$$



# Charge Density, Electric Field, Potential



For p-type

$$E(x) = -\frac{eN_a}{\epsilon_s}x + C$$
$$= -\frac{eN_a}{\epsilon_s}(x + x_p)$$

For n-type

$$E(x) = \frac{eN_d}{\epsilon_s}x + C$$
$$= \frac{eN_d}{\epsilon_s}(x - x_n)$$

$$C = \epsilon_r \epsilon_0 \frac{A}{d}$$
$$\frac{dE(x)}{dx} = \frac{\rho(x)}{\epsilon_s}$$

$\epsilon_s = \epsilon_r \epsilon_0$   
Permittivity [F/m]

Poisson's equation

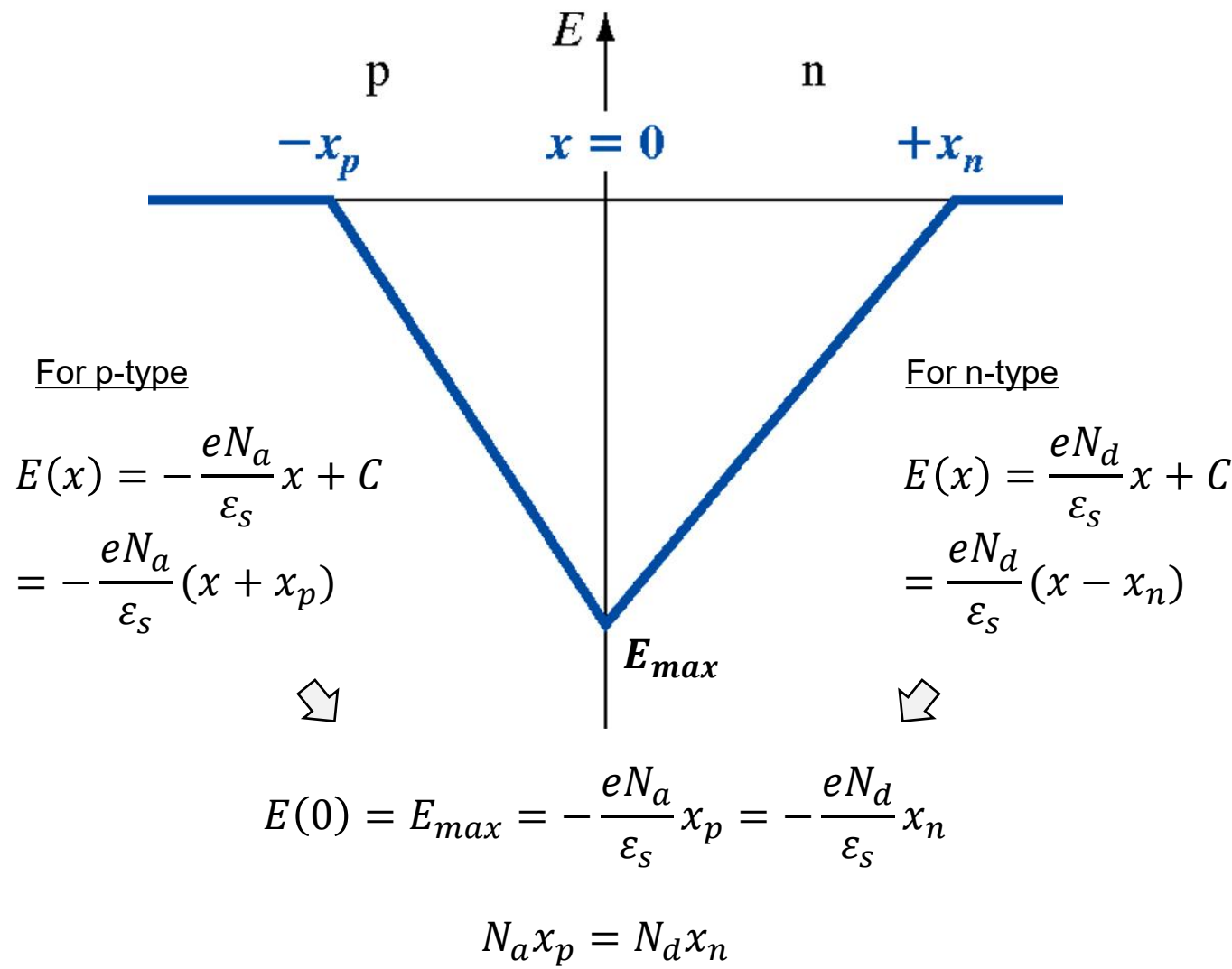
$$\frac{d^2\phi(x)}{dx^2} = -\frac{dE(x)}{dx} = -\frac{\rho(x)}{\epsilon_s}$$

$$\frac{dE(x)}{dx} = \frac{\rho(x)}{\epsilon_s} \Rightarrow E(x) = \int \frac{\rho(x)}{\epsilon_s} dx$$

Boundary conditions

$$E(-x_p) = 0$$
$$E(+x_n) = 0$$

# Charge Density, Electric Field, Potential



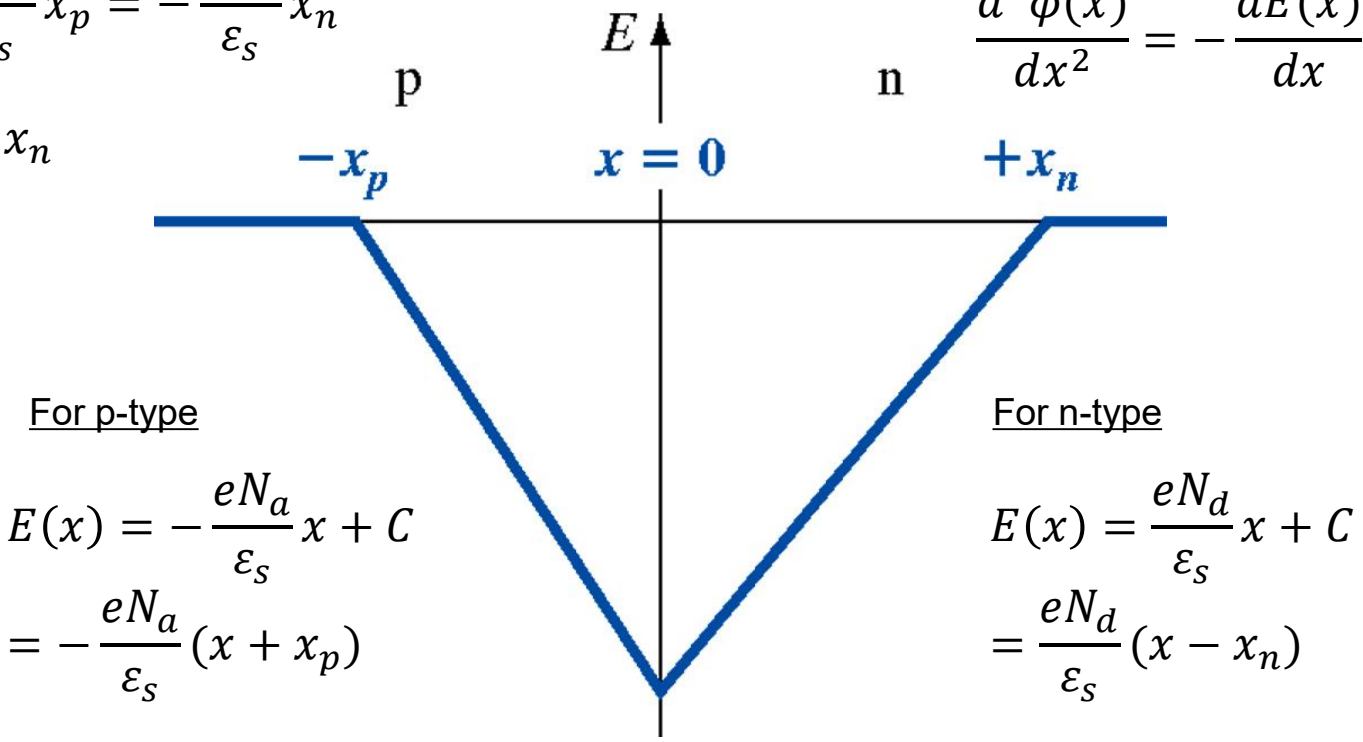
# Charge Density, Electric Field, Potential

$$E(0) = -\frac{eN_a}{\epsilon_s}x_p = -\frac{eN_d}{\epsilon_s}x_n$$

$$N_ax_p = N_dx_n$$

Poisson's equation

$$\frac{d^2\phi(x)}{dx^2} = -\frac{dE(x)}{dx} = -\frac{\rho(x)}{\epsilon_s}$$



$$\phi(x) = \frac{eN_a}{\epsilon_s} \int x + x_p dx \quad \Leftarrow \quad \phi(x) = - \int E(x) dx \quad \Rightarrow \quad \phi(x) = -\frac{eN_d}{\epsilon_s} \int x - x_n dx$$

$$= \frac{eN_a}{\epsilon_s} \left( \frac{1}{2} x^2 + x_p x \right) + C$$

$$= \frac{eN_a}{2\epsilon_s} (x + x_p)^2$$

Boundary conditions

$$\phi(-x_p) = 0$$

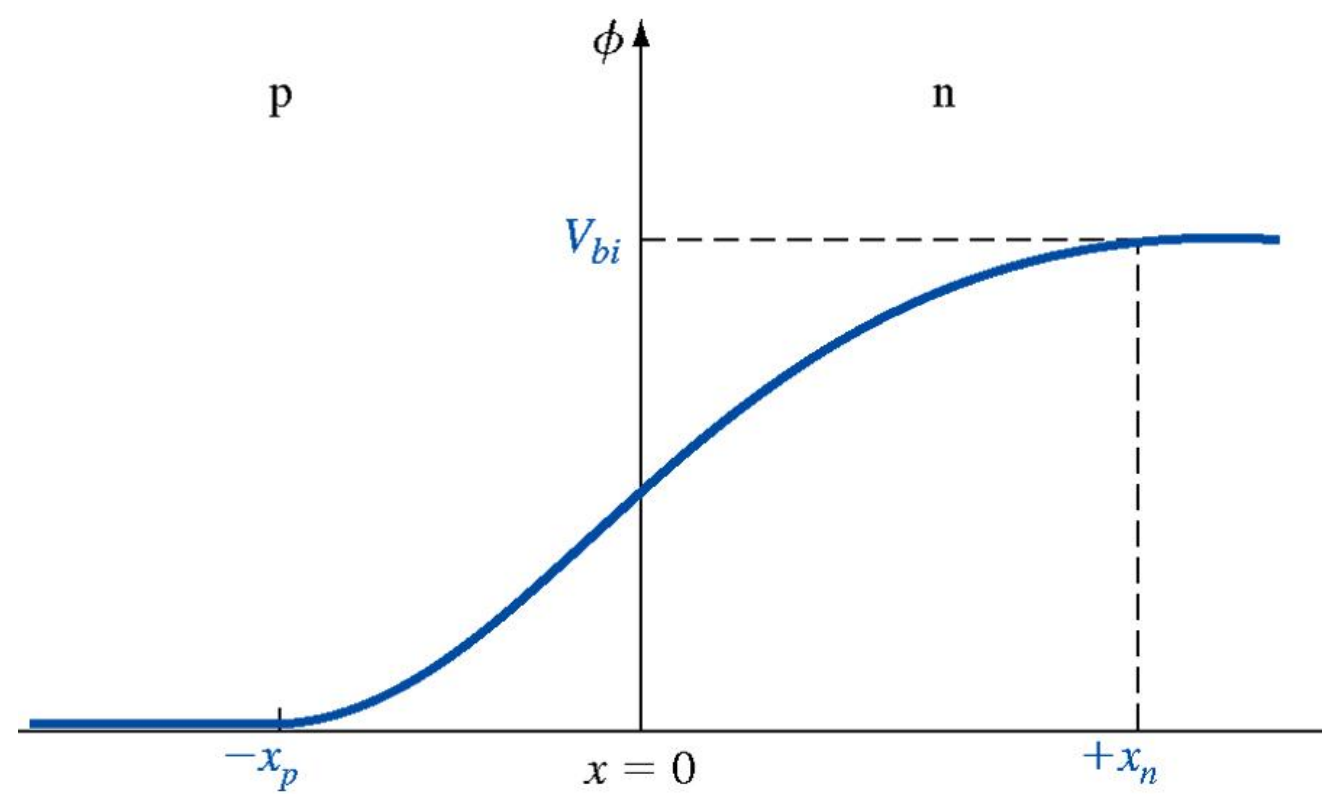
$$\phi(0) = \phi(0)$$

For p-type                      For n-type

$$= -\frac{eN_d}{\epsilon_s} \left( \frac{1}{2} x^2 - x_n x \right) + C$$

$$= -\frac{eN_d}{\epsilon_s} \left( \frac{1}{2} x^2 - x_n x \right) + \frac{eN_a}{2\epsilon_s} x_p^2$$

# Charge Density, Electric Field, Potential



$$\phi(x) = \frac{eN_a}{\epsilon_s} \int x + x_p dx \quad \Leftarrow \quad \phi(x) = - \int E(x) dx \quad \Rightarrow \quad \phi(x) = - \frac{eN_d}{\epsilon_s} \int x - x_n dx$$
$$= \frac{eN_a}{\epsilon_s} \left( \frac{1}{2} x^2 + x_p x \right) + C$$
$$= \frac{eN_a}{2\epsilon_s} (x + x_p)^2$$

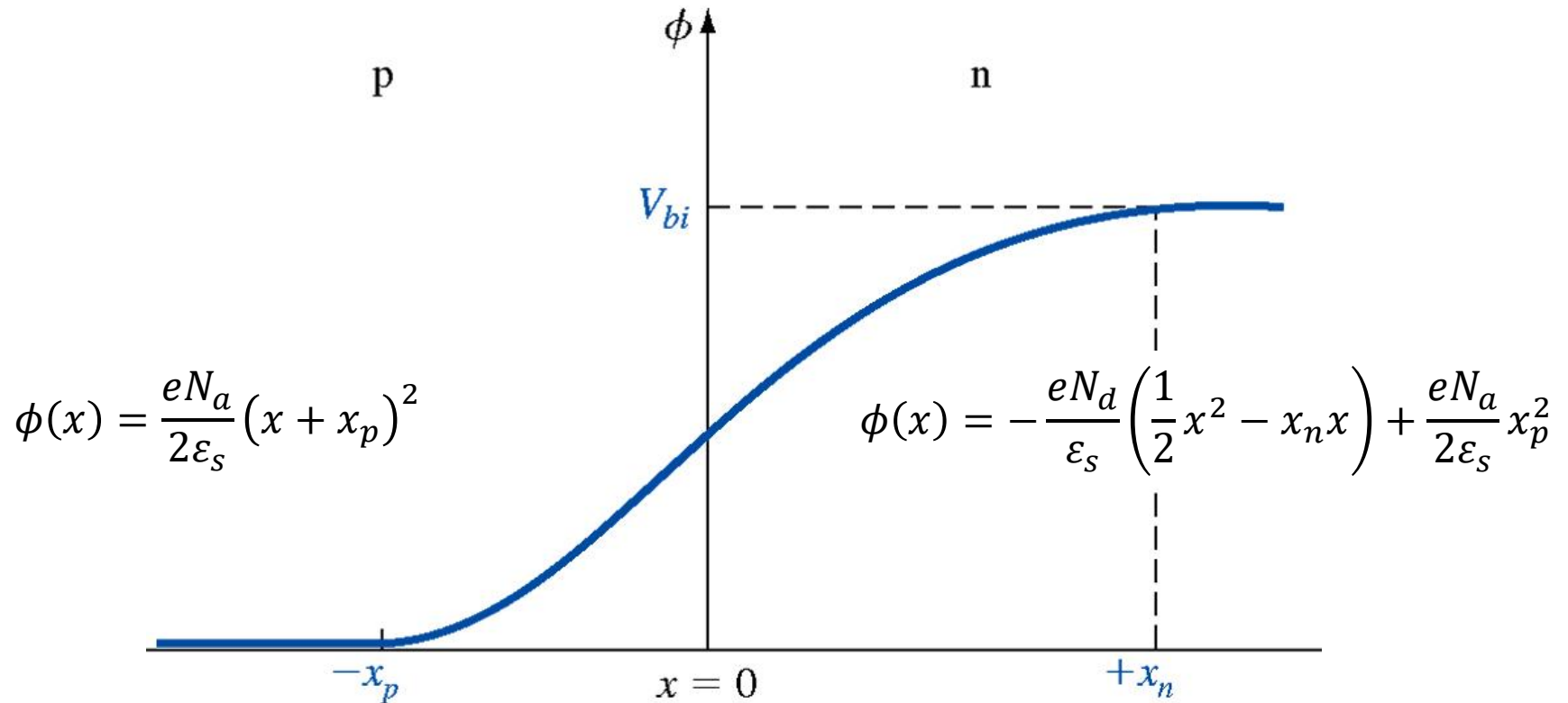
For p-type

$$\phi(0) =$$

For n-type

$$\phi(0) = - \frac{eN_a}{\epsilon_s} \left( \frac{1}{2} x^2 - x_n x \right) + C$$
$$= - \frac{eN_a}{\epsilon_s} \left( \frac{1}{2} x^2 - x_n x \right) + \frac{eN_a}{2\epsilon_s} x_p^2$$

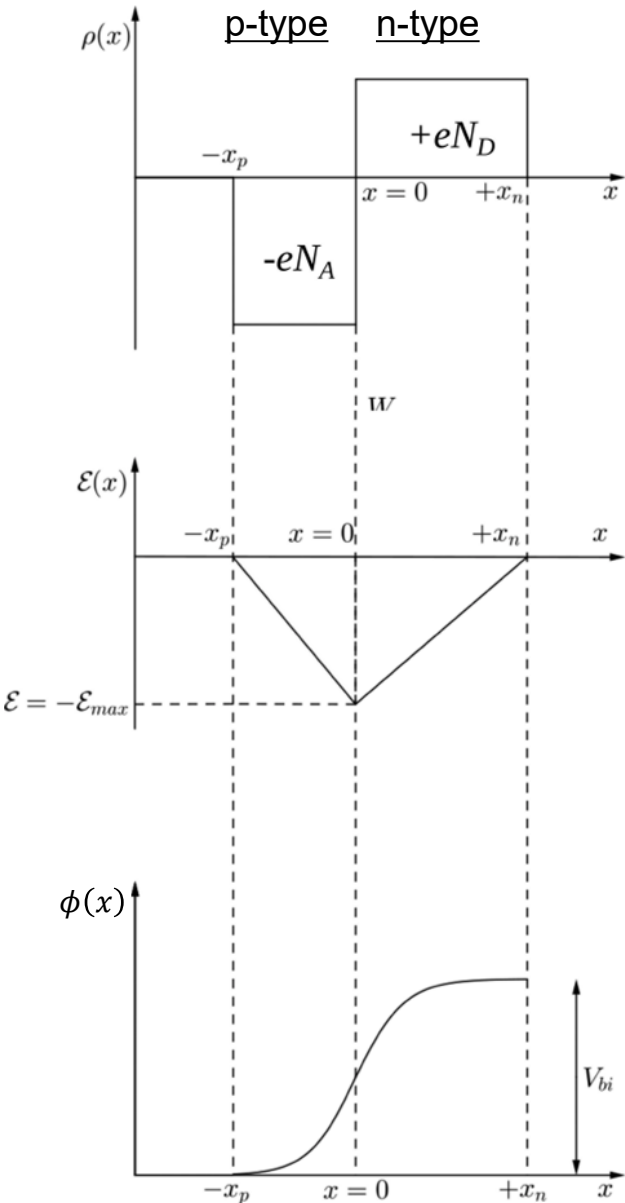
# Charge Density, Electric Field, Potential



$$\phi(x_n) = \frac{eN_d}{2\epsilon_s} x_n^2 + \frac{eN_a}{2\epsilon_s} x_p^2 = \frac{e}{2\epsilon_s} (N_d x_n^2 + N_a x_p^2)$$

$$\Rightarrow V_{bi} = \frac{e}{2\epsilon_s} (N_d x_n^2 + N_a x_p^2)$$

# Charge Density, Electric Field, Potential Summary



➤ **Poisson's equation**

$$\frac{d^2 \phi(x)}{dx^2} = - \frac{dE(x)}{dx} = - \frac{\rho(x)}{\epsilon_s}$$

➤ **Electric field**

For p-type  $E(x) = - \frac{eN_a}{\epsilon_s} (x + x_p)$

For n-type  $E(x) = \frac{eN_d}{\epsilon_s} (x - x_n)$

⇒  $N_a x_p = N_d x_n$

➤ **Potential**

For p-type  $\phi(x) = \frac{eN_a}{2\epsilon_s} (x + x_p)^2$

For n-type  $\phi(x) = - \frac{eN_d}{\epsilon_s} \left( \frac{1}{2} x^2 - x_n x \right) + \frac{eN_a}{2\epsilon_s} x_p^2$

⇒  $V_{bi} = \frac{e}{2\epsilon_s} (N_d x_n^2 + N_a x_p^2)$

# Space Charge Width

$$N_a x_p = N_d x_n$$

$$V_{bi} = \frac{e}{2\epsilon_s} (N_d x_n^2 + N_a x_p^2)$$

↓

$$x_p = \frac{N_d}{N_a} x_n$$

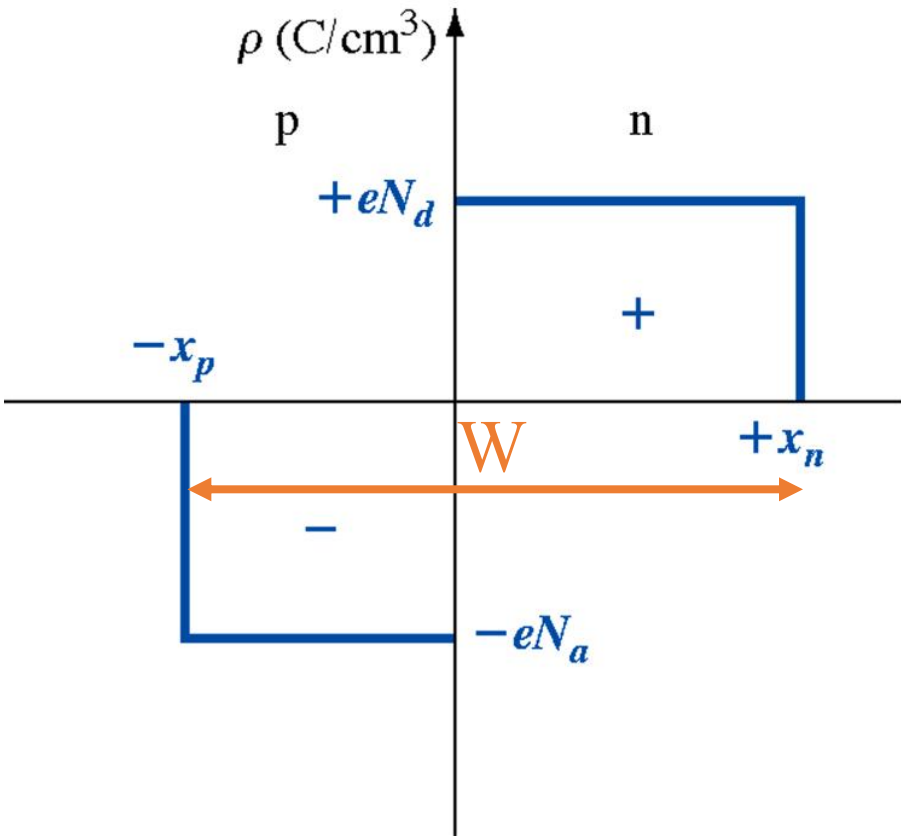
$$x_n = \left\{ \frac{2\epsilon_s V_{bi}}{e} \left[ \frac{N_a}{N_d} \right] \left[ \frac{1}{N_a + N_d} \right] \right\}^{1/2}$$

$$x_p = \left\{ \frac{2\epsilon_s V_{bi}}{e} \left[ \frac{N_d}{N_a} \right] \left[ \frac{1}{N_a + N_d} \right] \right\}^{1/2}$$

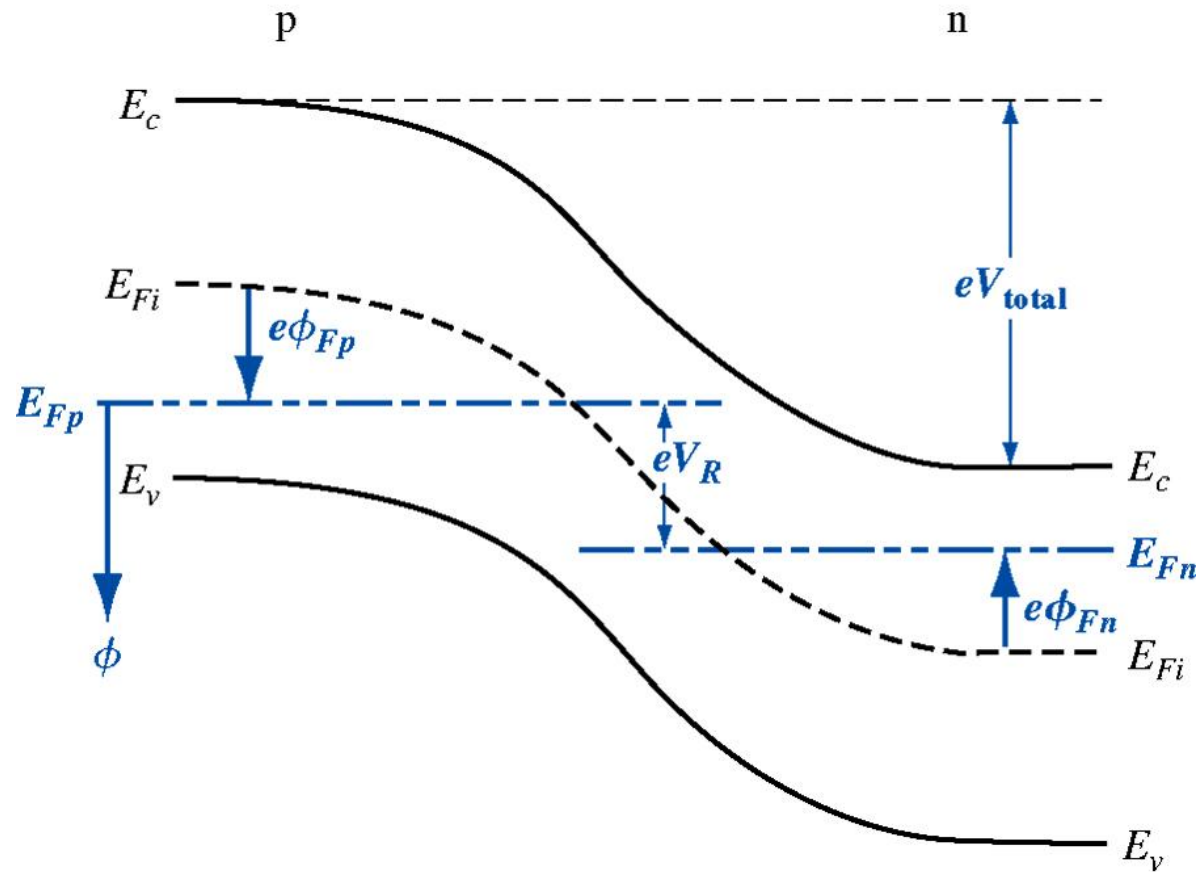


$$W = x_n + x_p = \left\{ \frac{2\epsilon_s V_{bi}}{e} \left[ \frac{N_a + N_d}{N_a N_d} \right] \right\}^{1/2} = \left\{ \frac{2\epsilon_s V_{bi}}{e} \left[ \frac{1}{N_a} + \frac{1}{N_d} \right] \right\}^{1/2}$$

$$x_n = W \left( \frac{N_a}{N_a + N_d} \right) \qquad x_p = W \left( \frac{N_d}{N_a + N_d} \right)$$



# Reverse Bias



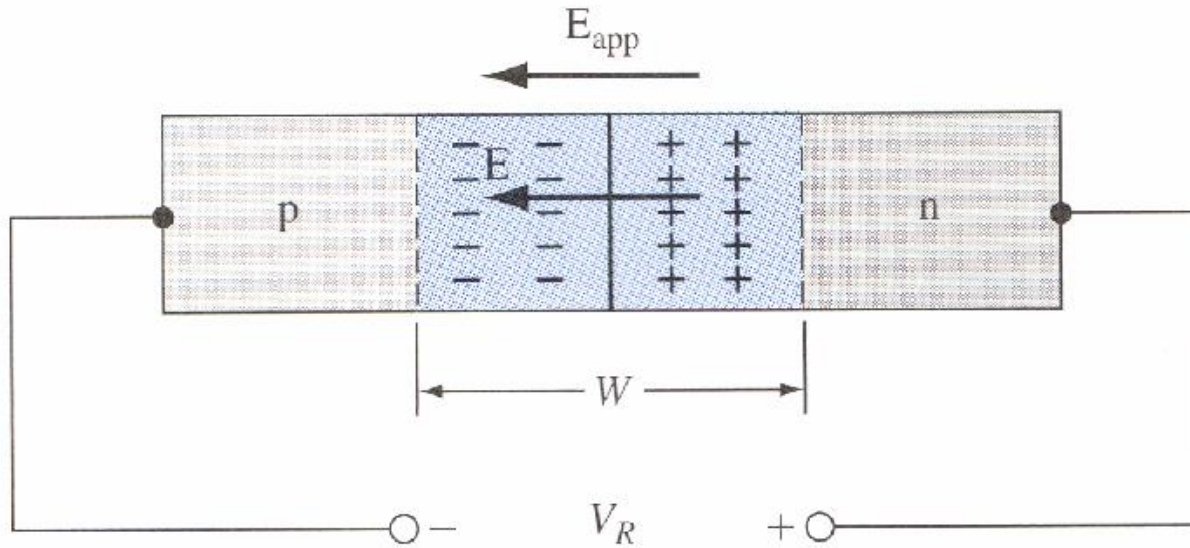
$$V_{total} = |\phi_{Fn}| + |\phi_{Fp}| = V_{bi}$$

⇓ Under application of Reverse bias

$$V_{total} = |\phi_{Fn}| + |\phi_{Fp}| + V_R = V_{bi} + V_R$$



## Reverse Bias (Space Charge Width)



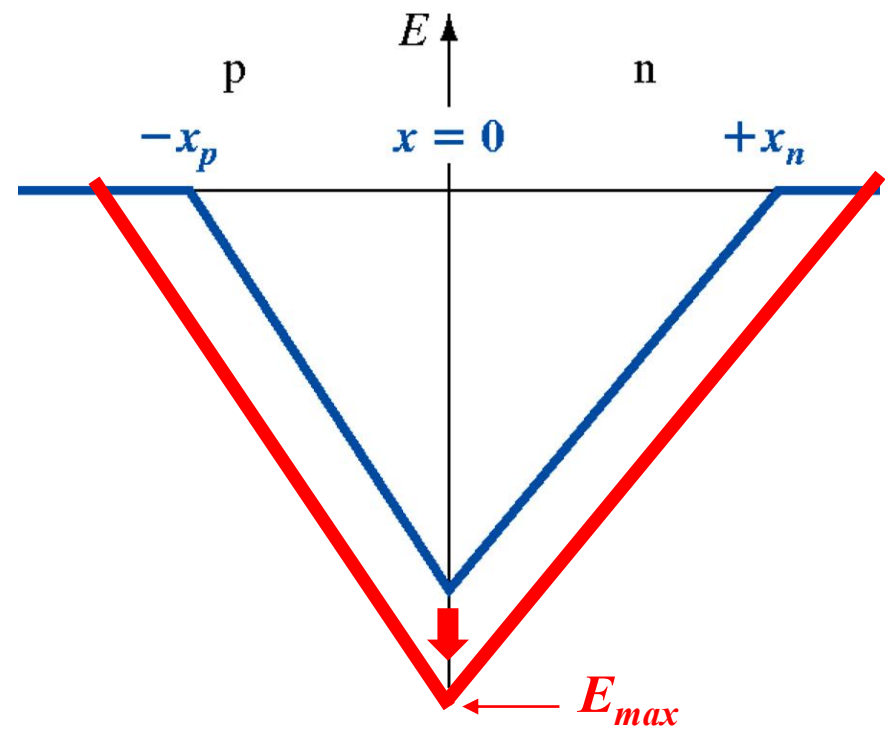
$$W = \left\{ \frac{2\epsilon_s V_{bi}}{e} \left[ \frac{N_a + N_d}{N_a N_d} \right] \right\}^{1/2}$$



Under application of Reverse bias

$$W = \left\{ \frac{2\epsilon_s (V_{bi} + V_R)}{e} \left[ \frac{N_a + N_d}{N_a N_d} \right] \right\}^{1/2}$$

# Reverse Bias (Electric Field)



$$E(0) = E_{max} = -\frac{eN_a}{\epsilon_s}x_p = -\frac{eN_d}{\epsilon_s}x_n$$

Under application of Reverse bias

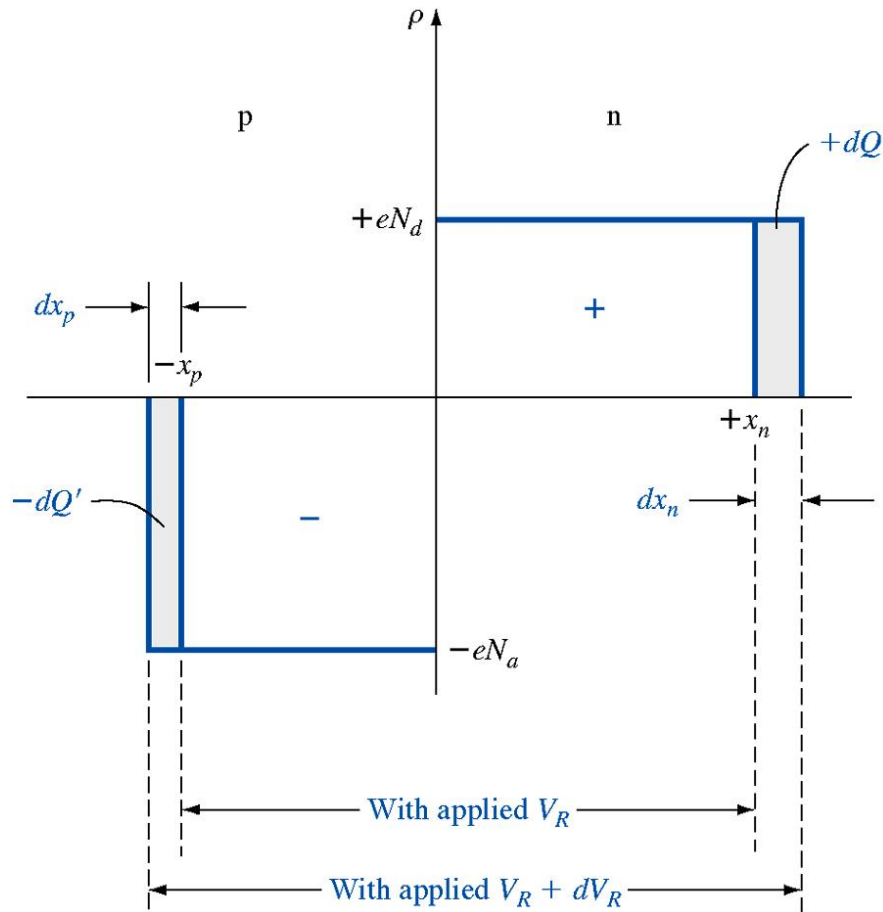
$$x_n = \left\{ \frac{2\epsilon_s(V_{bi} + V_R)}{e} \left[ \frac{N_a}{N_d} \right] \left[ \frac{1}{N_a + N_d} \right] \right\}^{1/2}$$

$$x_p = \left\{ \frac{2\epsilon_s(V_{bi} + V_R)}{e} \left[ \frac{N_d}{N_a} \right] \left[ \frac{1}{N_a + N_d} \right] \right\}^{1/2}$$

$$W = \left\{ \frac{2\epsilon_s(V_{bi} + V_R)}{e} \left[ \frac{N_a + N_d}{N_a N_d} \right] \right\}^{1/2}$$

$$\begin{aligned} E_{max} &= -\frac{eN_a}{\epsilon_s} \left\{ \frac{2\epsilon_s(V_{bi} + V_R)}{e} \left[ \frac{N_d}{N_a} \right] \left[ \frac{1}{N_a + N_d} \right] \right\}^{1/2} = -\left\{ \frac{2e(V_{bi} + V_R)}{\epsilon_s} \left[ \frac{N_a N_d}{N_a + N_d} \right] \right\}^{1/2} \\ &= -\left\{ \frac{2^2 e (V_{bi} + V_R)^2}{2\epsilon_s (V_{bi} + V_R)} \left[ \frac{N_a N_d}{N_a + N_d} \right] \right\}^{1/2} = -2(V_{bi} + V_R) \left\{ \frac{e}{2\epsilon_s (V_{bi} + V_R)} \left[ \frac{N_a N_d}{N_a + N_d} \right] \right\}^{1/2} \\ &= \frac{-2(V_{bi} + V_R)}{W} \end{aligned}$$

# Junction Capacitance



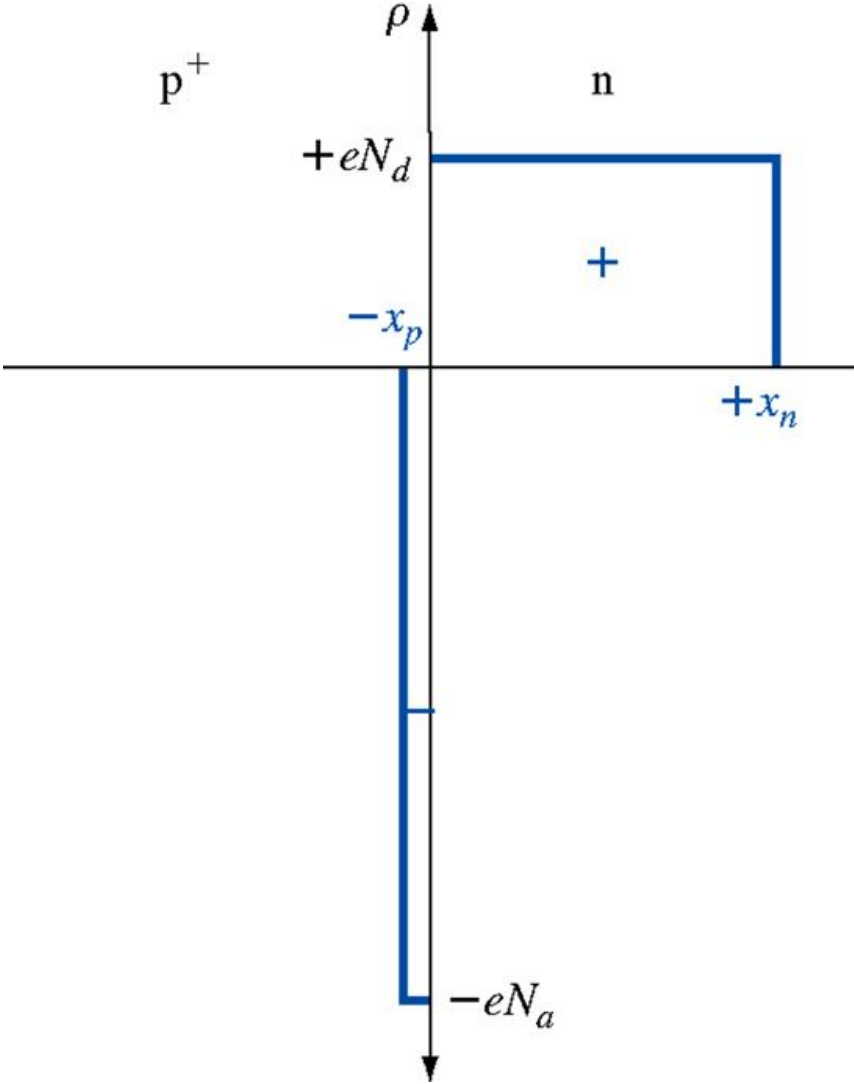
$$x_n = W \left( \frac{N_a}{N_a + N_d} \right)$$

$$\frac{1}{W} = \frac{1}{x_n} \left( \frac{N_a}{N_a + N_d} \right)$$

$$x_n = \left\{ \frac{2\epsilon_s(V_{bi} + V_R)}{e} \left[ \frac{N_a}{N_d} \right] \left[ \frac{1}{N_a + N_d} \right] \right\}^{1/2}$$

$$\begin{aligned} C' &= \frac{dQ'}{dV_R} = \frac{deN_dx_n}{dV_R} = eN_d \frac{dx_n}{dV_R} = \left\{ \frac{e\epsilon_s N_a N_d}{2(V_{bi} + V_R)(N_a + N_d)} \right\}^{1/2} \\ &= \frac{eN_d}{2x_n} \left( \frac{2\epsilon_s}{e} \left[ \frac{N_a}{N_d} \right] \left[ \frac{1}{N_a + N_d} \right] \right) = \frac{\epsilon_s}{x_n} \left( \frac{N_a}{N_a + N_d} \right) = \frac{\epsilon_s}{W} [F/cm^2] \end{aligned}$$

# One-Sided Junction



when  $p \gg n$   $\Rightarrow N_a \gg N_d$

$$W = \left\{ \frac{2\epsilon_s(V_{bi} + V_R)}{e} \left[ \frac{N_a + N_d}{N_a N_d} \right] \right\}^{1/2}$$

$\Downarrow$  Divide by  $N_a$

$$W \approx \left\{ \frac{2\epsilon_s(V_{bi} + V_R)}{e N_d} \right\}^{1/2} \approx x_n$$

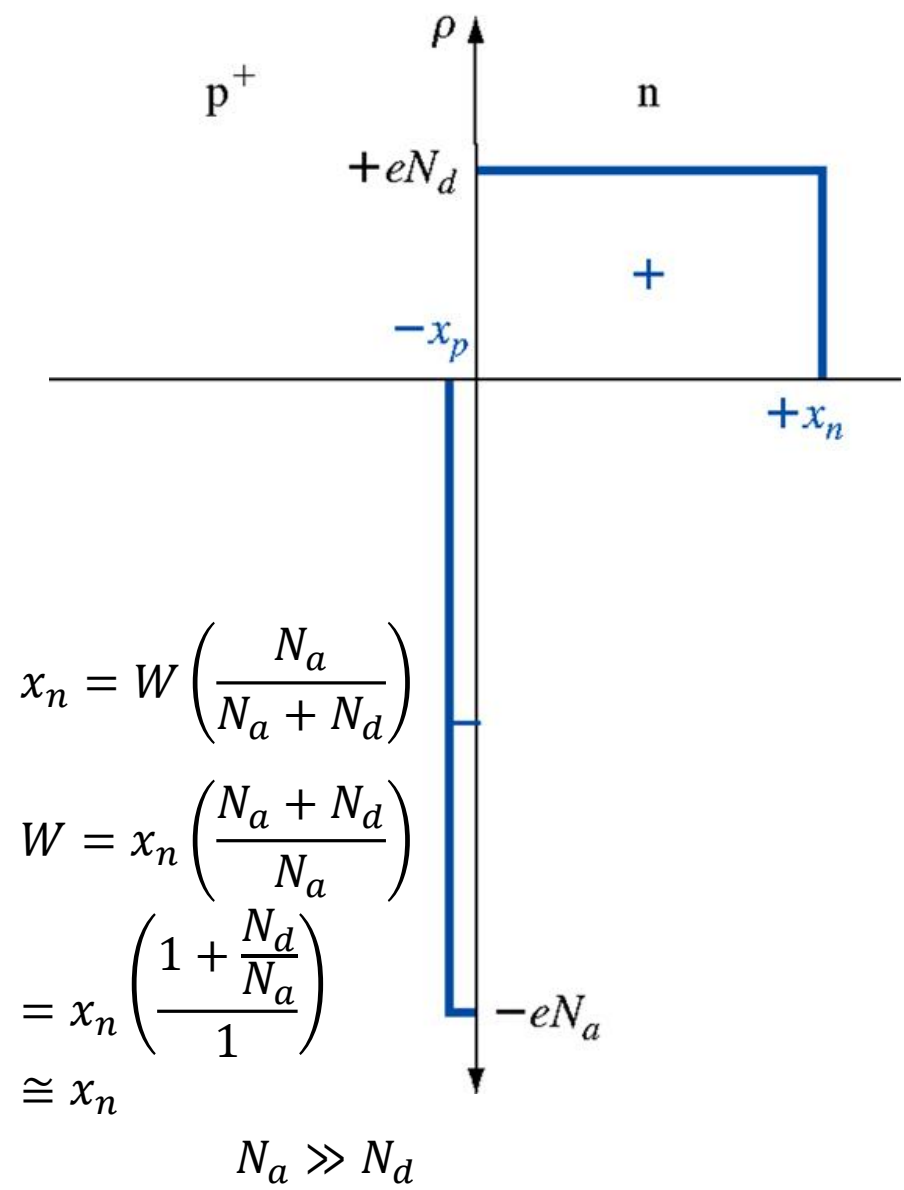
$\Uparrow$

$$x_n = \left\{ \frac{2\epsilon_s(V_{bi} + V_R)}{e} \left[ \frac{N_a}{N_d} \right] \left[ \frac{1}{N_a + N_d} \right] \right\}^{1/2}$$

$\Downarrow$  Divide by  $N_a$

$$x_n \approx \left\{ \frac{2\epsilon_s(V_{bi} + V_R)}{e N_d} \right\}^{1/2}$$

# One-Sided Junction



$when\ p \gg\ n \Rightarrow N_a \gg N_d$

$$W = \left\{ \frac{2\epsilon_s(V_{bi} + V_R)}{e} \left[ \frac{N_a + N_d}{N_a N_d} \right] \right\}^{1/2}$$

⇓ Divide by  $N_a$

$$W \approx \left\{ \frac{2\epsilon_s(V_{bi} + V_R)}{e N_d} \right\}^{1/2} \approx x_n$$

⇑

$$x_n = \left\{ \frac{2\epsilon_s(V_{bi} + V_R)}{e} \left[ \frac{N_a}{N_d} \right] \left[ \frac{1}{N_a + N_d} \right] \right\}^{1/2}$$

⇓ Divide by  $N_a$

$$x_n \approx \left\{ \frac{2\epsilon_s(V_{bi} + V_R)}{e N_d} \right\}^{1/2}$$

# One-Sided Junction Capacitance

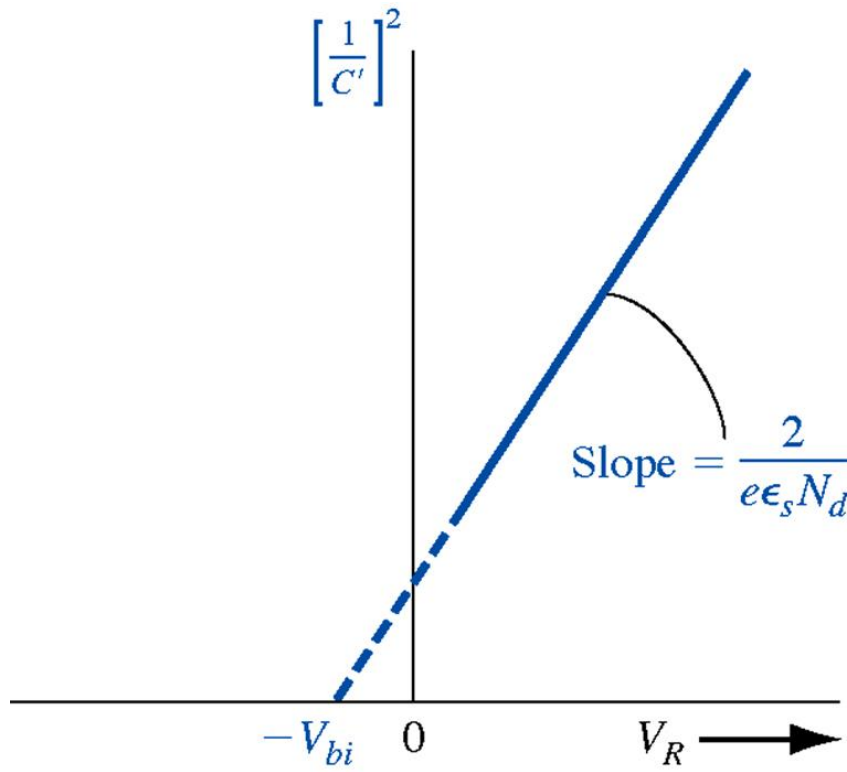
$$C' = eN_d \frac{dx_n}{dV_R} = \left\{ \frac{e\epsilon_s N_a N_d}{2(V_{bi} + V_R)(N_a + N_d)} \right\}^{1/2} = \frac{\epsilon_s}{x_n} \left( \frac{N_a}{N_a + N_d} \right)$$

⇓ Divide by  $N_a$

$$C' = eN_d \frac{dx_n}{dV_R} = \left\{ \frac{e\epsilon_s N_d}{2(V_{bi} + V_R)} \right\}^{1/2} = \frac{\epsilon_s}{x_n}$$

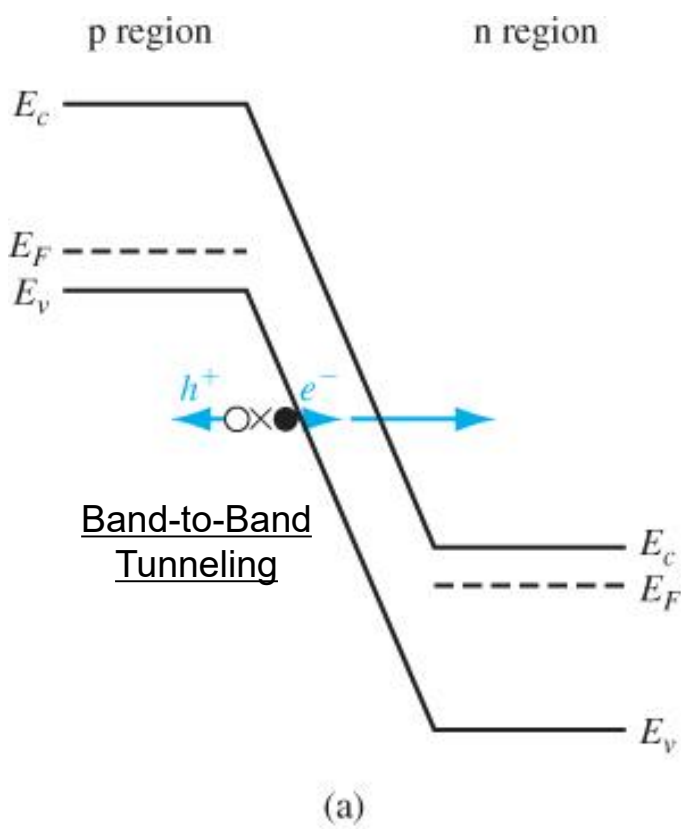
⇓

$$\left[ \frac{1}{C'} \right]^2 = \frac{2}{e\epsilon_s N_d} (V_R + V_{bi})$$

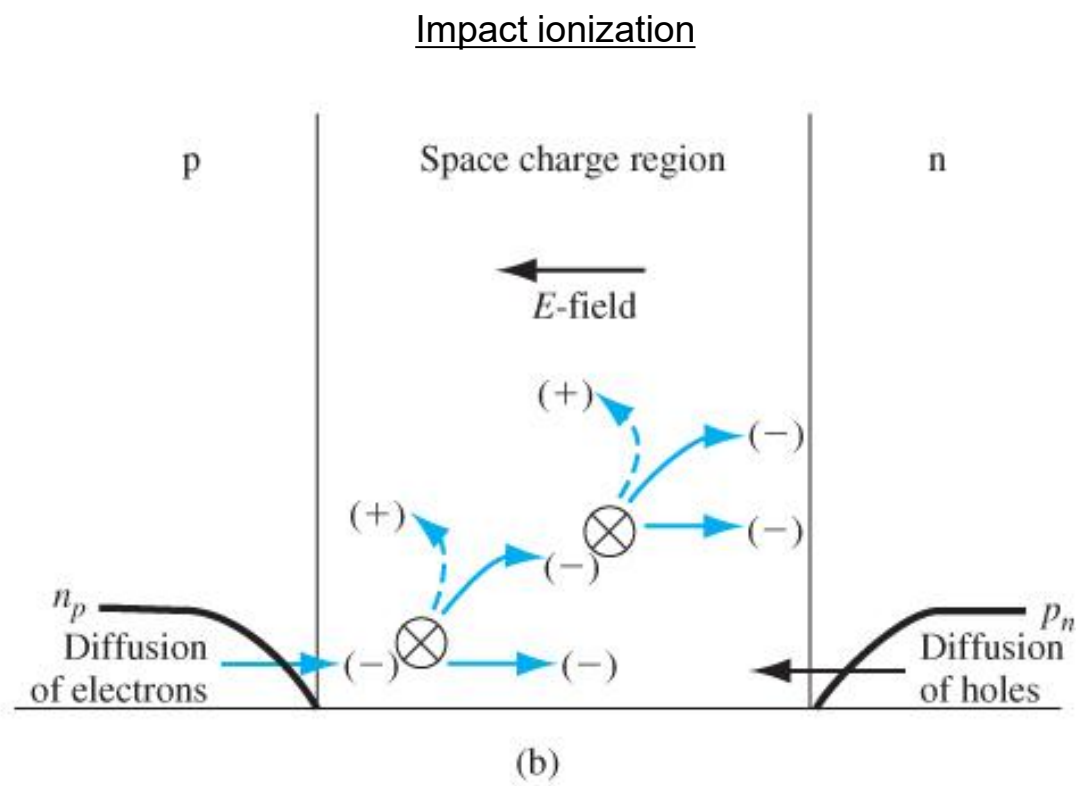


# PN Junction Breakdown in Reverse Bias

Zener breakdown

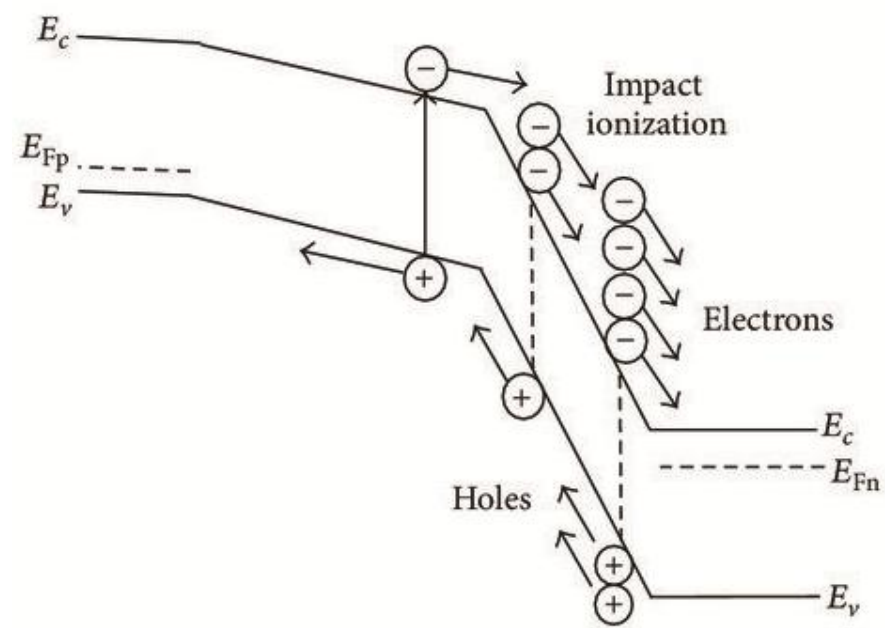
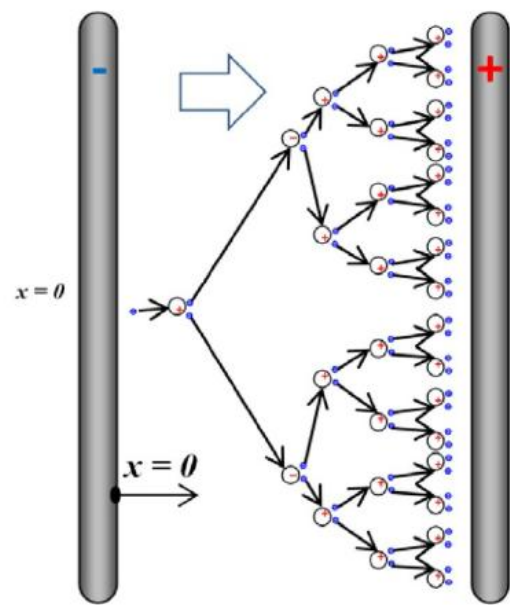
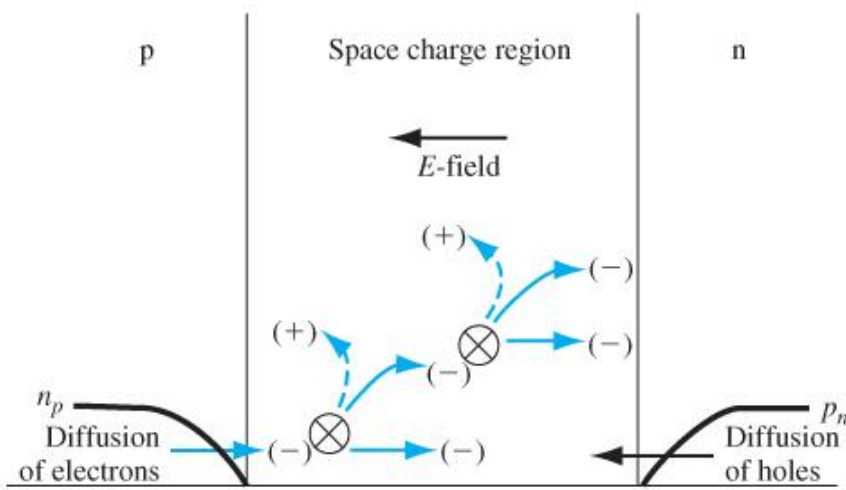


Avalanche breakdown



# PN Junction Breakdown in Reverse Bias

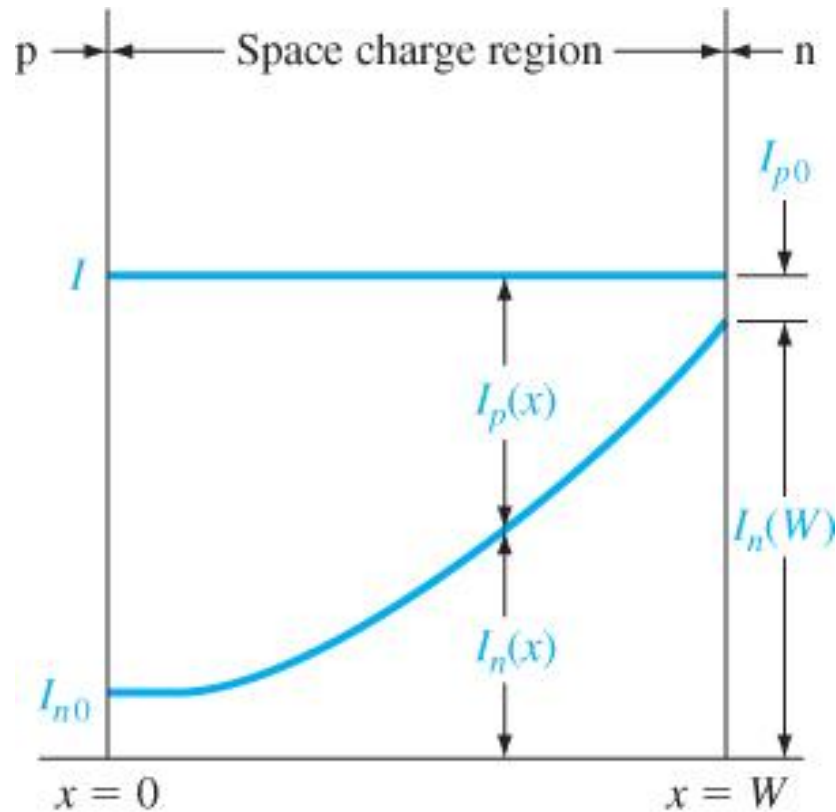
## Avalanche breakdown





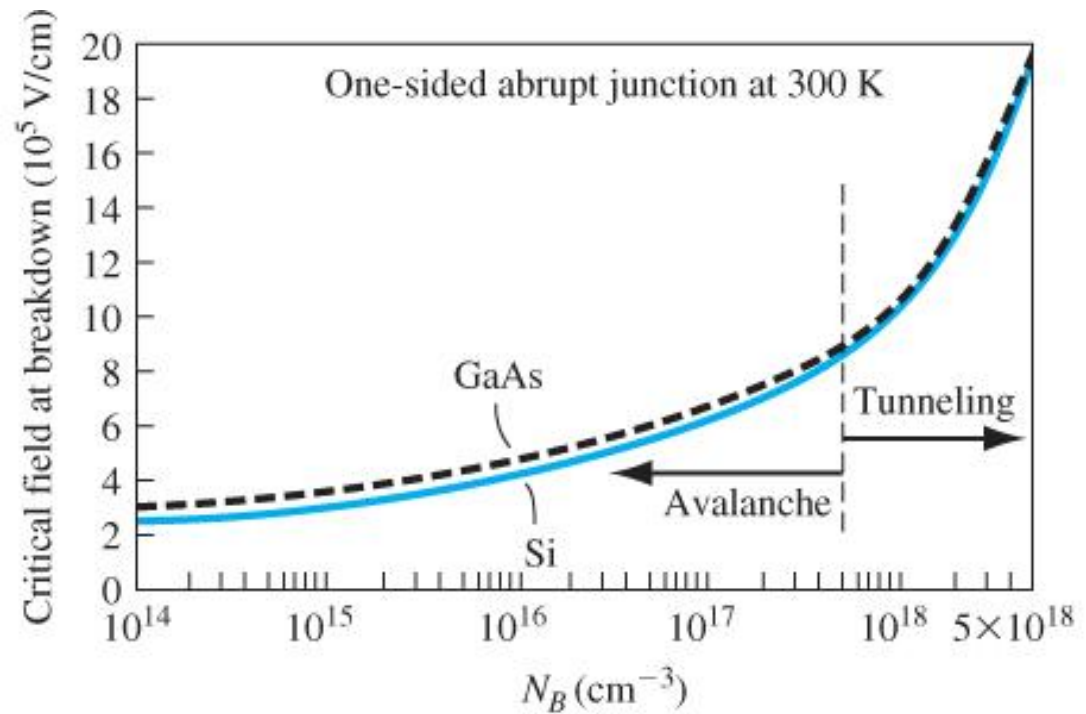
# PN Junction Breakdown in Reverse Bias

## Avalanche breakdown



**Figure 7.13** | Electron and hole current components through the space charge region during avalanche multiplication.

# PN Junction Breakdown in Reverse Bias



$$E_{max} \cong -\frac{eN_d}{\epsilon_s} x_n$$
$$x_n \approx \left\{ \frac{2\epsilon_s(V_{bi} + V_R)}{eN_d} \right\}^{1/2}$$
$$\approx \left\{ \frac{2\epsilon_s V_R}{eN_d} \right\}^{1/2} \quad V_R \gg V_{bi}$$

Assume

$|E_{max}| = E_{crit}$  when  $V_R = V_B$  in  $N_B$  doping concentration

**Critical electric field**                      **Breakdown voltage**

$$E_{crit} \cong \frac{eN_B}{\epsilon_s} \left\{ \frac{2\epsilon_s V_B}{eN_B} \right\}^{1/2} = \left\{ \frac{2eN_B V_B}{\epsilon_s} \right\}^{1/2} \Rightarrow V_B = \frac{\epsilon_s E_{crit}^2}{2eN_B}$$