Semiconductor Devices

Chapter 6
Nonequilibrium Excess Carriers in Semiconductors

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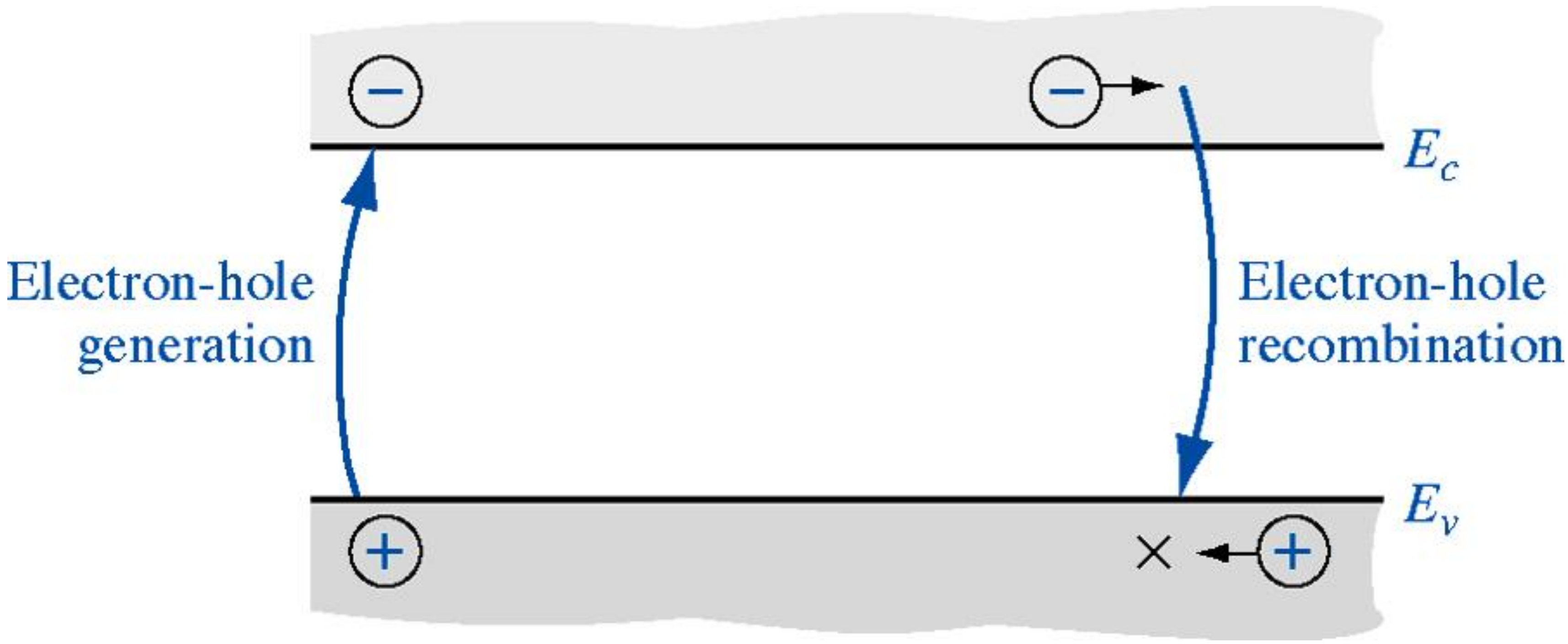
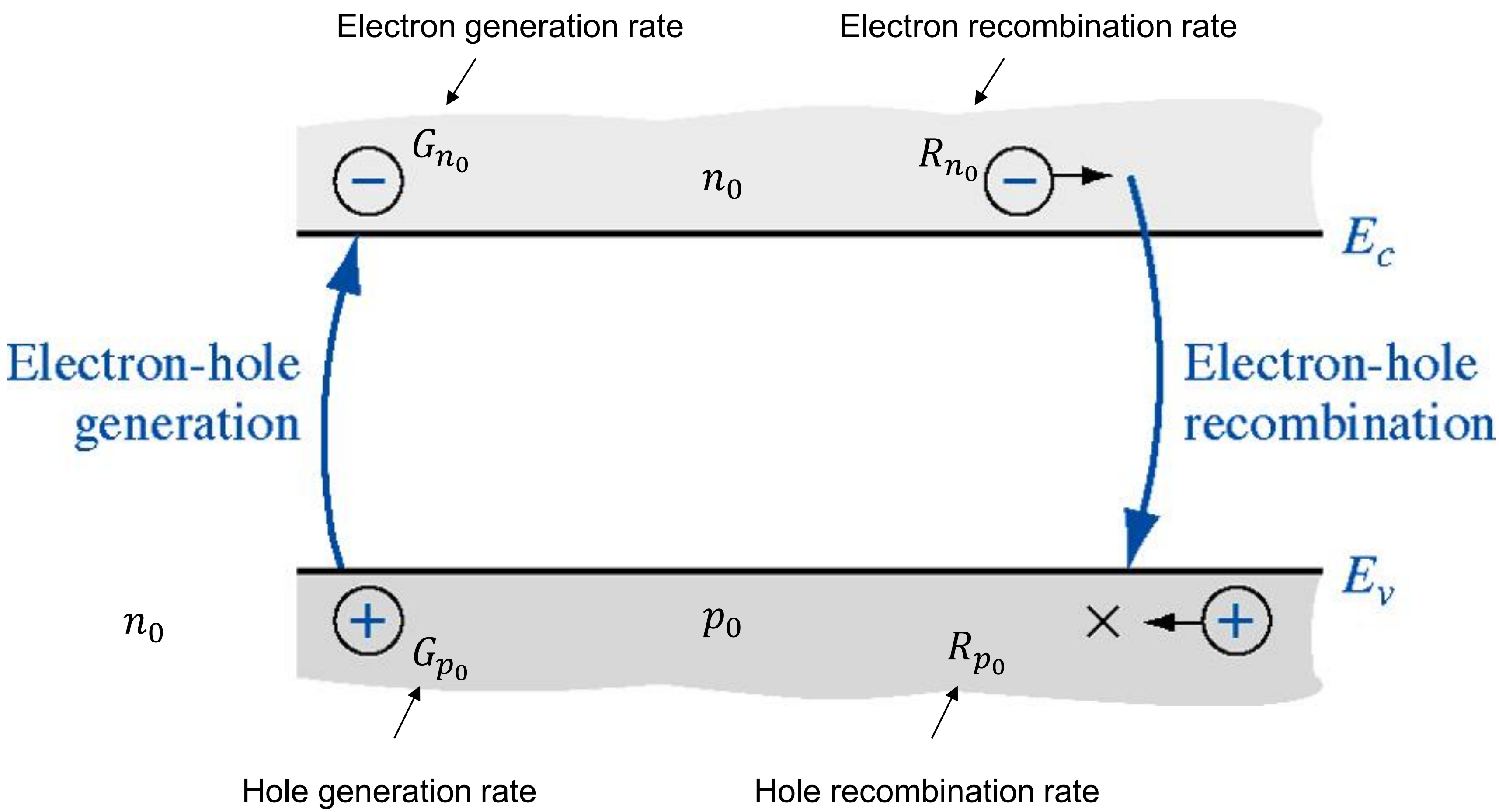
Thermal Equilibrium State



- ✓ **Generation: produces an Electron-Hole pair**
- ✓ **Recombination: eliminates an Electron-Hole pair**

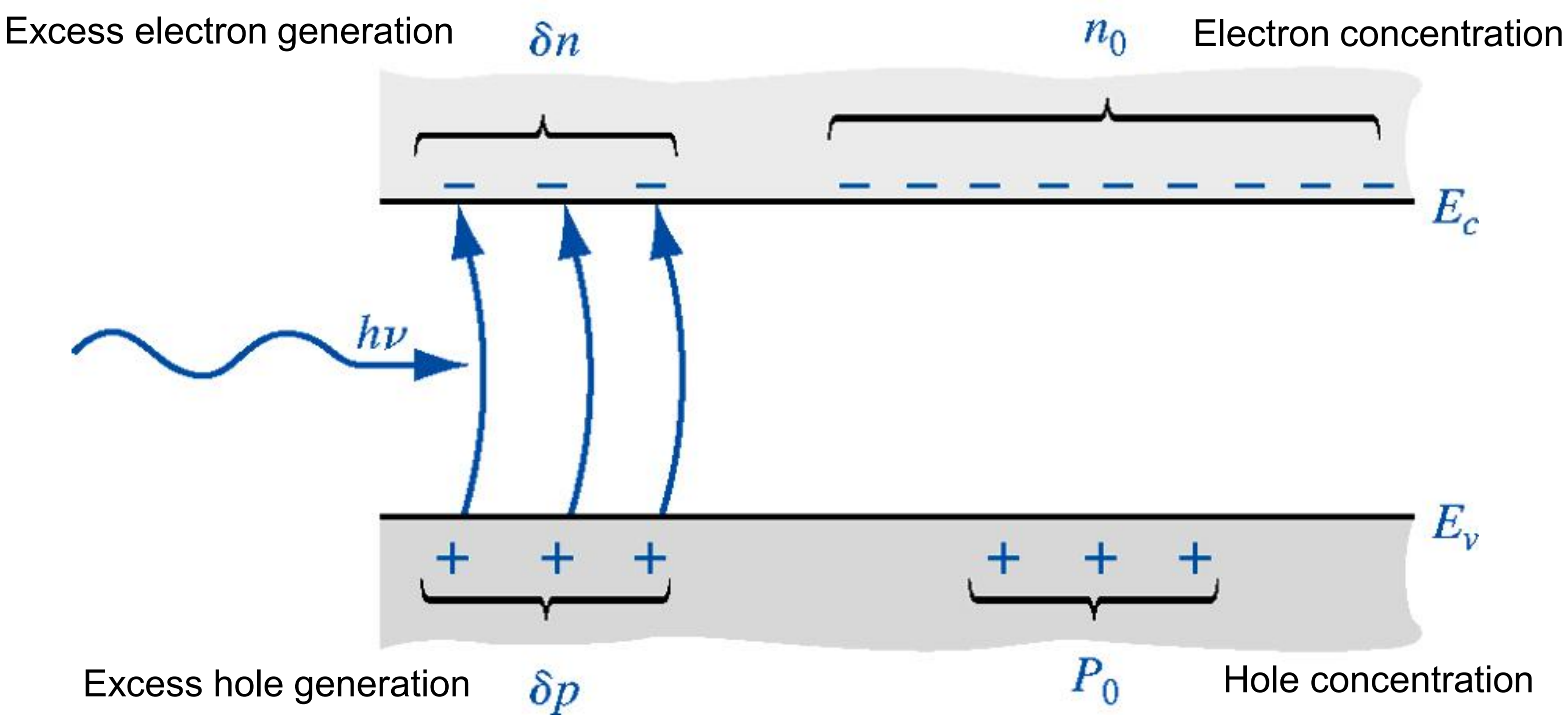
Thermal Equilibrium State



$$G_{n_0} = G_{p_0} = R_{n_0} = R_{p_0}\ [\#/\mathrm{cm}^3 \cdot \mathrm{s}]$$ (In equilibrium state)

$$n_0 p_0 = n_i^2$$

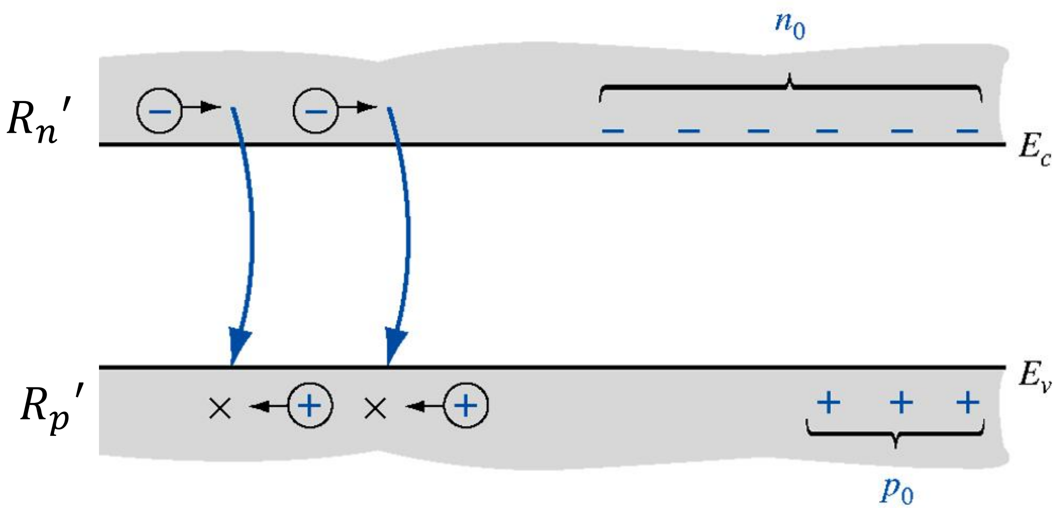
Nonequilibrium State



$$n = n_0 + \delta n, \qquad p = p_0 + \delta p \quad \Rightarrow \quad np \neq n_0 p_0 = n_i{}^2$$

Mass-action law

Recombination Rate of Excess Carriers



$$n = n_0 + \delta n \qquad p = p_0 + \delta p$$

$$np \neq n_0 p_0 = {n_i}^2$$

Mass-action law

Excess carrier recombination rate

$$R_n' = \frac{\partial(\delta n(t,x))}{\partial t} \qquad R_p' = \frac{\partial(\delta p(t,x))}{\partial t} \qquad [\#/\mathrm{cm}^3 \cdot \mathrm{s}]$$

⇩ simplify

τ_R **Recombination carrier lifetime**

→ Mean time for excess carrier recombination

$$R_n' \approx \frac{\delta n}{\tau_R} \qquad R_p' \approx \frac{\delta p}{\tau_R}$$

⇩

For n-type Si

$$R_n' = \frac{\delta n}{\boldsymbol{\tau_p}} = R_p' = \frac{\delta p}{\boldsymbol{\tau_p}}$$

Excess minority carrier lifetime

For p-type Si

$$R_n' = \frac{\delta n}{\boldsymbol{\tau_n}} = R_p' = \frac{\delta p}{\boldsymbol{\tau_n}}$$

Generation Rate of Excess Carriers

Excess carrier generation rate

$$g_n{}' = \frac{\delta n}{\tau_G} \qquad g_p{}' = \frac{\delta p}{\tau_G} \qquad [\#/\mathrm{cm}^3 \cdot \mathrm{s}]$$

Generation carrier lifetime
→ Mean time for excess carrier generation

$$g_n{}' = g_p{}' = constant$$ (In general cases)

Steady State Condition

Steady-state

$$\frac{dn}{dt} = \frac{dp}{dt} = 0$$

$$\frac{dn}{dt} = \left(G_{n_0} + g_n{}'\right) - \left(R_{n_0} + R_n'\right) = 0 \qquad g_n{}' = R_n{}'$$

$$\frac{dp}{dt} = \left(G_{p_0} + g_p{}'\right) - \left(R_{p_0} + R_p'\right) = 0 \qquad g_p{}' = R_p{}'$$

Steady-state
→ Regardless of Nonequilibrium or Equilibrium but no net changes in concentrations

Table 6.1 | Relevant notation used in Chapter 6

Symbol	Definition
n_0, p_0	Thermal equilibrium electron and hole concentrations (independent of time and also usually position).
n, p	Total electron and hole concentrations (may be functions of time and/or position).
$\delta n = n - n_0$ $\delta p = p - p_0$	Excess electron and hole concentrations (may be functions of time and/or position).
g'_n, g'_p	Excess electron and hole generation rates.
R'_n, R'_p	Excess electron and hole recombination rates.
τ_{n0}, τ_{p0}	Excess minority carrier electron and hole lifetimes.

Quasi-Fermi Energy Level for Low Level Injection

For n-type Si

$n_0 \gg p_0, \qquad n_0 \gg |\delta n| = |\delta p| \gg p_0$ ⇨ $n = n_0 + \delta n \cong n_0$

$p = p_0 + \delta p \cong \boldsymbol{\delta p}$

For p-type Si

$p_0 \gg n_0, \qquad p_0 \gg |\delta n| = |\delta p| \gg n_0$ ⇨ $p = p_0 + \delta p \cong p_0$

$n = n_0 + \delta n \cong \boldsymbol{\delta n}$

❖ **Quasi-Fermi Energy Level**

$$n_0 + \delta n = N_c \exp\left(-\frac{E_c - E_{Fn}}{kT}\right) = n_i \exp\left(\frac{E_{Fn} - E_{Fi}}{kT}\right)$$

$$p_0 + \delta p = N_v \exp\left(-\frac{E_{Fp} - E_v}{kT}\right) = n_i \exp\left(\frac{E_{Fi} - E_{Fp}}{kT}\right)$$

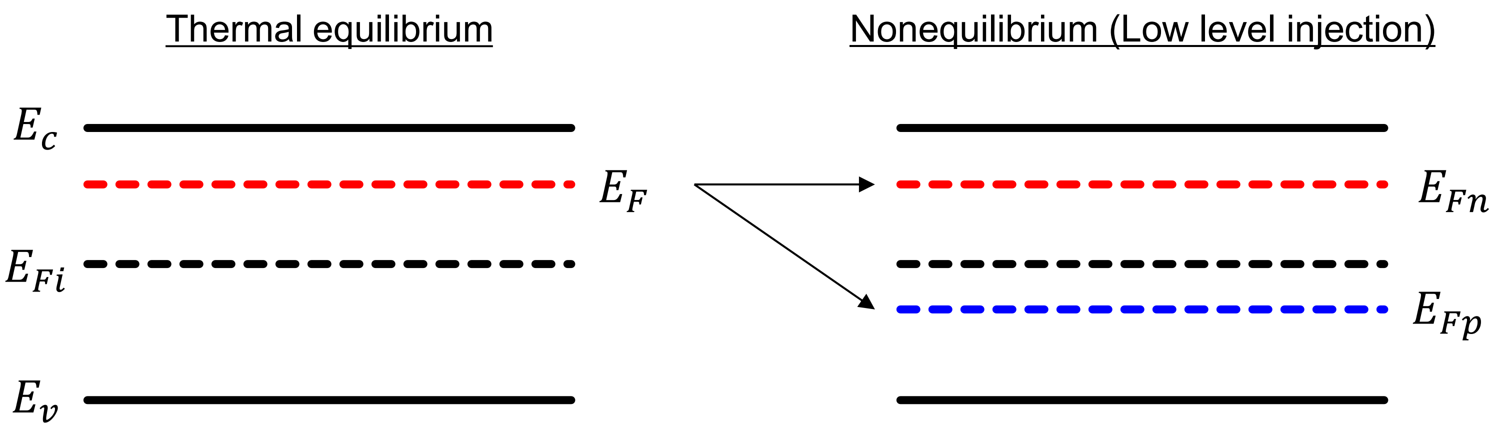
Quasi-Fermi Energy Level for Low Level Injection

❖ Quasi-Fermi Energy Level

For n-type Si

$$n_0 + \delta n = N_c \exp\left(-\frac{E_c - E_{Fn}}{kT}\right) = n_i \exp\left(\frac{E_{Fn} - E_{Fi}}{kT}\right) \cong n_0$$

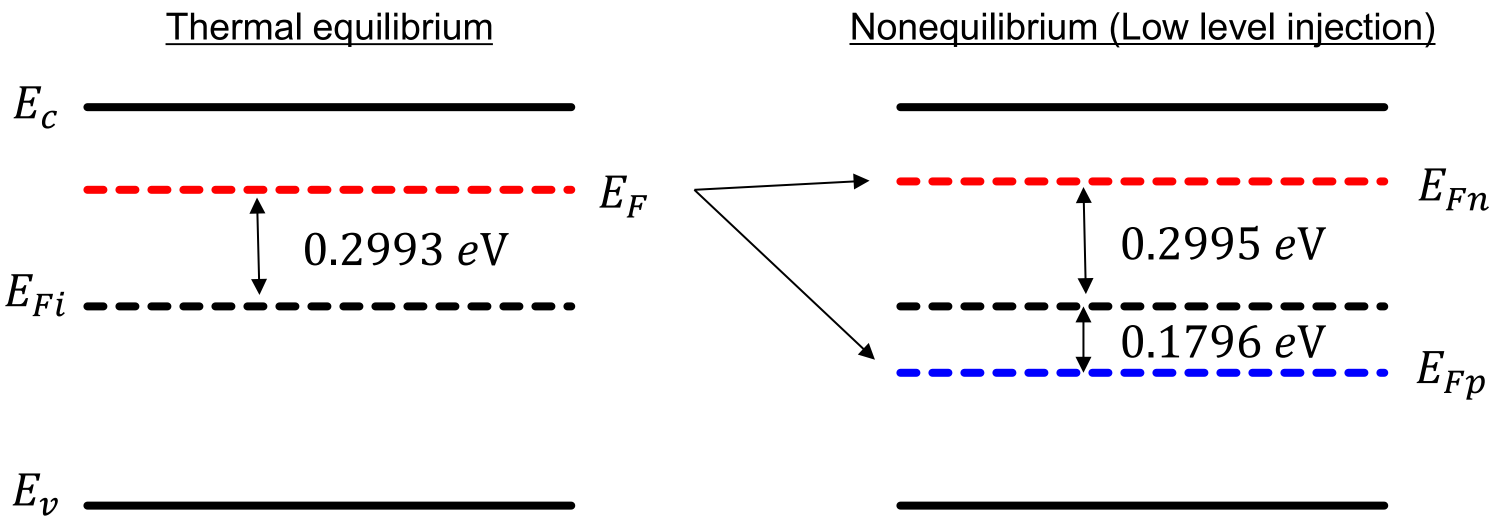
$$p_0 + \delta p = N_v \exp\left(-\frac{E_{Fp} - E_v}{kT}\right) = n_i \exp\left(\frac{E_{Fi} - E_{Fp}}{kT}\right) \cong \delta n$$



Quasi-Fermi Energy Level for Low Level Injection (Example)

Find the energy difference between the intrinsic fermi level (E_{Fi}) and the quasi-fermi energy level (E_{Fn}, E_{Fp}) for low level injection. Assume a steady-state condition.

At $T = 300\ \mathrm{K}$, $N_d = 10^{15}\ \mathrm{cm}^{-3}$, $n_i = 10^{10}\ \mathrm{cm}^{-3}$, $\tau_p = 1\ \mu s$, $g_n' = \mathbf{10^{19}}\ \mathrm{cm}^{-3} s^{-1}$,
$kT = 0.026\ e\mathrm{V}$, $N_c = 2.8 \times 10^{19}\ \mathrm{cm}^{-3}$, $N_v = 1.04 \times 10^{19}\ \mathrm{cm}^{-3}$



Carrier Transport Mechanisms

- Drift: Due to the Electric field

- Diffusion: Due to the Carrier concetration gradient

- Recombination-Generation (R-G): Due to the Excess carriers

Continuity Equation

→ carrier concentration change per time at the volume of dxdydz

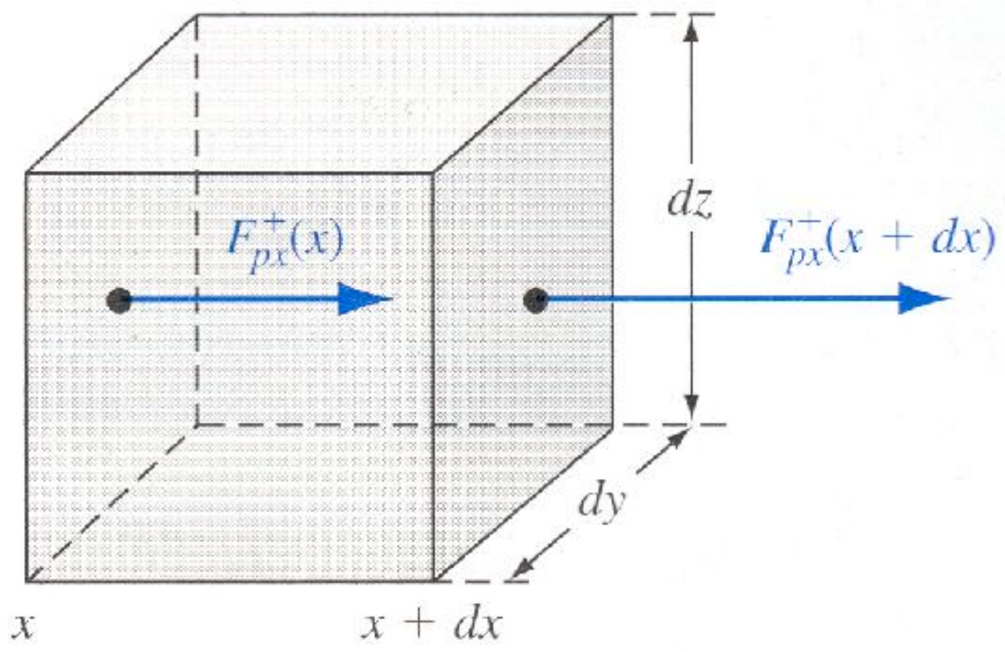
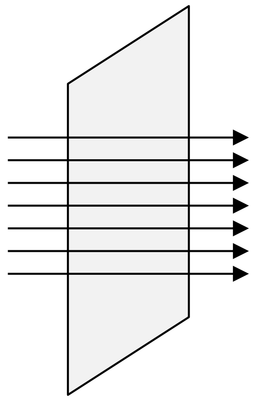


Figure 6.4 | Differential volume showing x component of the hole-particle flux.

Flux



$$Flux\ (F)[\#/\text{cm}^2 \cdot s] = nv\ [(\#/\text{cm}^3)(\text{cm}/s)]$$

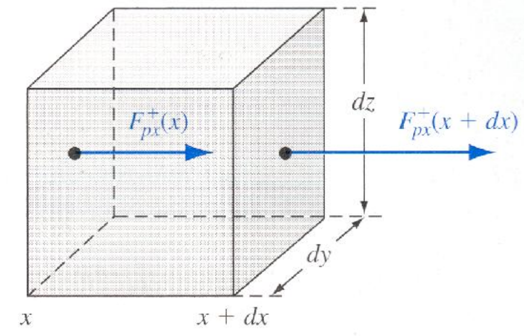


Figure 6.4 | Differential volume showing x component of the hole-particle flux.

$$J = qnv = qF$$

Flux

$$J_n = J_{n|drf} + J_{n|diff} = q\boxed{n\mu_n E} + q\boxed{D_n \frac{dn}{dx}}$$

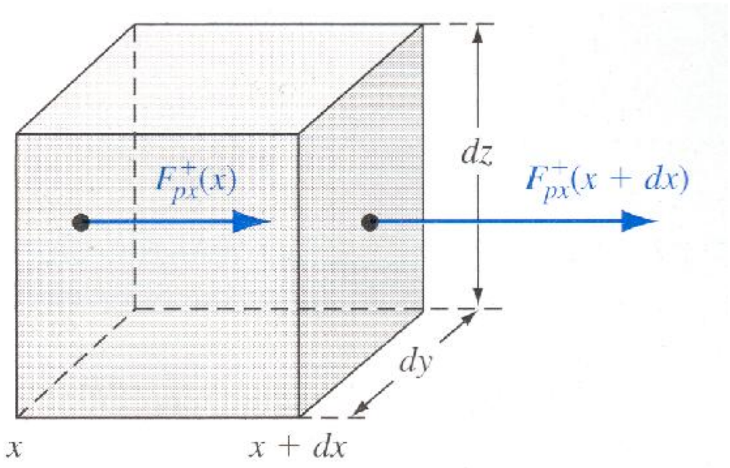
$$J_p = J_{p|drf} + J_{p|diff} = q\boxed{p\mu_p E} - q\boxed{D_p \frac{dp}{dx}}$$

Flux

Continuity Equation

Continuity Equation

→ carrier concentration change per time at the volume of dxdydz



$$\frac{\partial p}{\partial t} dxdydz = [F(x) - F(x + dx)]dydz = -\frac{F(x + dx) - F(x)}{dx} dxdydz$$

$$= -\frac{\partial F(x)}{\partial x} dxdydz$$

$$\frac{\partial p}{\partial t} = -\frac{\partial F(x)}{\partial x}$$

Consider R-G

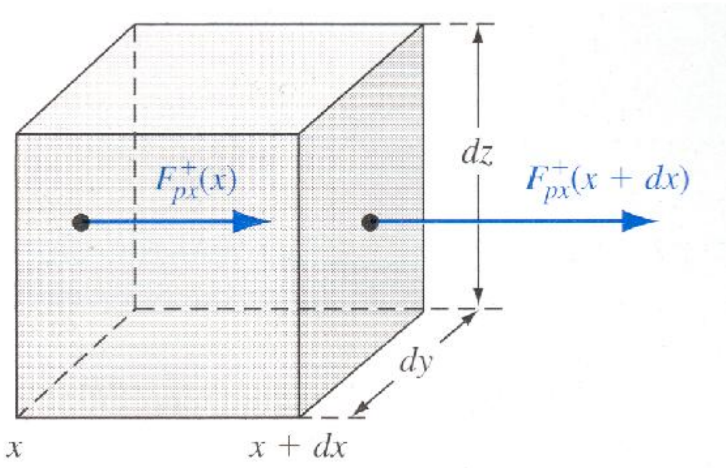
⇨

$$\frac{\partial p}{\partial t} = -\frac{\partial F(x)}{\partial x} + G_p - R_p$$

Continuity Equation

Continuity Equation

→ carrier concentration change per time at the volume of dxdydz



$$\frac{\partial p}{\partial t} = -\frac{\partial F(x)}{\partial x} + G_p - R_p$$

$$J = qnv = qF(x) \qquad F(x) = \frac{1}{q}J$$

$$\frac{\partial p}{\partial t} = -\frac{\partial F(x)}{\partial x} + G_p - R_p$$

Continuity Equation

$$\frac{\partial p(t,x)}{\partial t} = -\frac{1}{q}\frac{\partial J_p}{\partial x} + g_p{}' - R_p{}' \qquad R_p{}' = \frac{\delta p}{\tau_p}$$

$$\frac{\partial n(t,x)}{\partial t} = +\frac{1}{q}\frac{\partial J_n}{\partial x} + g_n{}' - R_n{}' \qquad R_n{}' = \frac{\delta n}{\tau_n}$$

⇦

$$J_p = qp\mu_p E - qD_p\frac{dp}{dx}$$

$$J_n = qn\mu_n E + qD_n\frac{dn}{dx}$$

Continuity Equation

Continuity Equation
→ carrier concentration change per time at the volume of dxdydz

$$\frac{\partial p(t,x)}{\partial t} = -\frac{1}{q}\frac{\partial J_p}{\partial x} + g_p{}' - R_p{}' \qquad R_p{}' = \frac{\delta p}{\tau_p}$$

$$\frac{\partial n(t,x)}{\partial t} = +\frac{1}{q}\frac{\partial J_n}{\partial x} + g_n{}' - R_n{}' \qquad R_n{}' = \frac{\delta n}{\tau_n}$$

$$J_p(t,x) = qp(t,x)\mu_p E(x) - qD_p\frac{\partial p(t,x)}{\partial x}$$

$$J_n(t,x) = qn(t,x)\mu_n E(x) + qD_n\frac{\partial n(t,x)}{\partial x}$$

$$p(t,x) = p_0(x) + \delta p(t,x)$$

$$n(t,x) = n_0(x) + \delta n(t,x)$$

Assume uniform doping

$$p(t,x) = p_0 + \delta p(t,x)$$

$$n(t,x) = n_0 + \delta n(t,x)$$

For Holes

$$\frac{\partial \delta p(t,x)}{\partial t} = D_p\frac{\partial^2 \delta p(t,x)}{\partial x^2} - \mu_p\left(\frac{\partial \delta p(t,x)}{\partial x}E(x) + p(t,x)\frac{\partial E(x)}{\partial x}\right) + g_p{}' - \frac{\delta p(t,x)}{\tau_p}$$

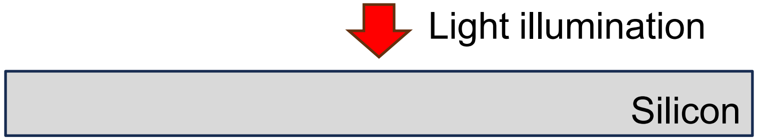
For Electrons

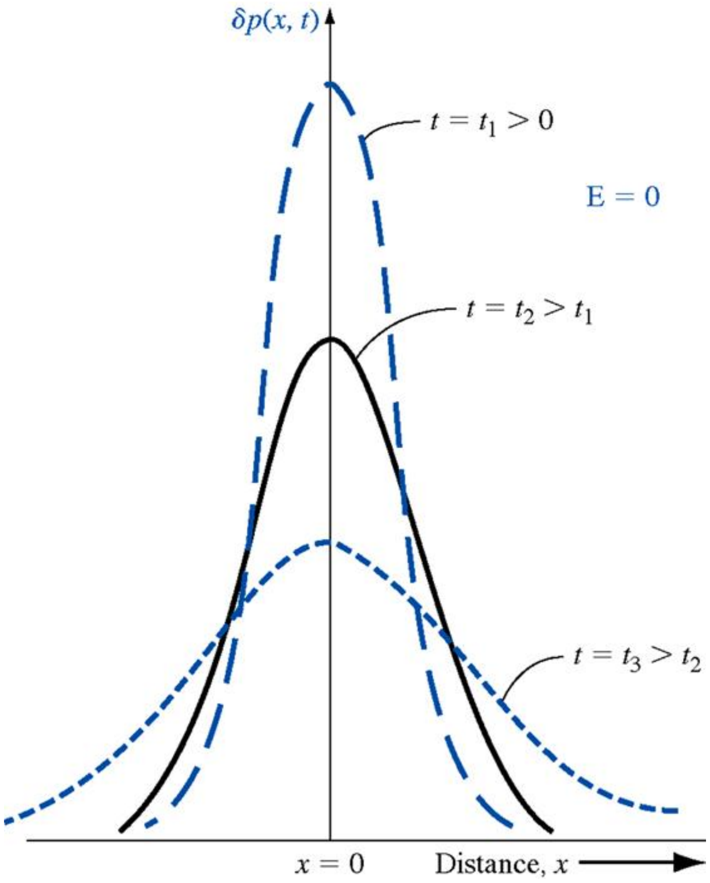
$$\frac{\partial \delta n(t,x)}{\partial t} = D_n\frac{\partial^2 \delta n(t,x)}{\partial x^2} + \mu_n\left(\frac{\partial \delta n(t,x)}{\partial x}E(x) + n(t,x)\frac{\partial E(x)}{\partial x}\right) + g_n{}' - \frac{\delta n(t,x)}{\tau_n}$$

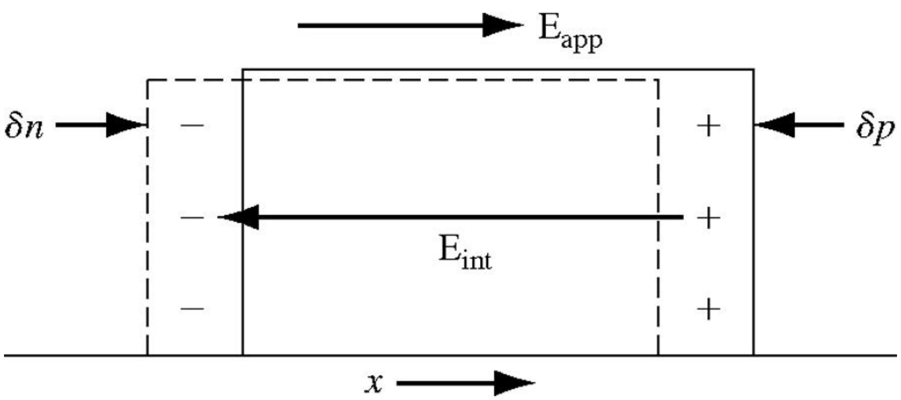
Ambipolar Transport

Ambipolar transport

→ excess electron and hole move together as a pair → Single Mobility and Single Diffusion constant







$$E = E_{app} + E_{int}$$

Ambipolar Transport

Assume

$$\delta p(t,x) = \delta n(t,x) \qquad g_p{}' = g_n{}' = g' \qquad \frac{\delta p(t,x)}{\tau_p} = \frac{\delta n(t,x)}{\tau_n} = R'$$

$$\frac{\partial \delta p(t,x)}{\partial t} = D_p \frac{\partial^2 \delta p(t,x)}{\partial x^2} - \mu_p \left(\frac{\partial \delta p(t,x)}{\partial x} E(x) + p(t,x) \frac{\partial E(x)}{\partial x} \right) + g_p{}' - \frac{\delta p(t,x)}{\tau_p}$$

$\times\, n(t,x)\mu_n$ $\qquad$ $\times\, n(t,x)\mu_n$

$$\frac{\partial \delta n(t,x)}{\partial t} = D_n \frac{\partial^2 \delta n(t,x)}{\partial x^2} + \mu_n \left(\frac{\partial \delta n(t,x)}{\partial x} E(x) + n(t,x) \frac{\partial E(x)}{\partial x} \right) + g_n{}' - \frac{\delta n(t,x)}{\tau_n}$$

$\times\, p(t,x)\mu_p$ $\qquad$ $\times\, p(t,x)\mu_p$

$\Downarrow$ Get rid of $\dfrac{\partial E(x)}{\partial x}$ $\qquad \delta N = \delta p = \delta n$

$$(n\mu_n + p\mu_p)\frac{\partial \delta N}{\partial t}$$

$$= (n\mu_n D_p + p\mu_p D_n)\frac{\partial^2 \delta N}{\partial x^2} + \mu_n \mu_p (p-n)\frac{\partial \delta N}{\partial x} E + (n\mu_n + p\mu_p)(g' - R')$$

Ambipolar Transport

$$(n\mu_n + p\mu_p)\frac{\partial \delta N}{\partial t}$$

$$= (n\mu_n D_p + p\mu_p D_n)\frac{\partial^2 \delta N}{\partial x^2} + \mu_n \mu_p (p-n)\frac{\partial \delta N}{\partial x}E + (n\mu_n + p\mu_p)(g' - R')$$

⇩ Devide by $n\mu_n + p\mu_p$

$$\frac{\partial \delta N}{\partial t} = \left(\frac{n\mu_n D_p + p\mu_p D_n}{n\mu_n + p\mu_p}\right)\frac{\partial^2 \delta N}{\partial x^2} + \left(\frac{\mu_n \mu_p (p-n)}{n\mu_n + p\mu_p}\right)\frac{\partial \delta N}{\partial x}E + g' - R'$$

⇩

$$\frac{\partial \delta N}{\partial t} = \boldsymbol{D}'\frac{\partial^2 \delta N}{\partial x^2} + \boldsymbol{\mu}'\frac{\partial \delta N}{\partial x}E + g' - R'$$

$$\frac{\partial \delta p(t,x)}{\partial t} = D_p \frac{\partial^2 \delta p(t,x)}{\partial x^2} - \mu_p\left(\frac{\partial \delta p(t,x)}{\partial x}E(x) + p(t,x)\frac{\partial E(x)}{\partial x}\right) + g_p{}' - \frac{\delta p(t,x)}{\tau_p}$$

$$\frac{\partial \delta n(t,x)}{\partial t} = D_n \frac{\partial^2 \delta n(t,x)}{\partial x^2} + \mu_n\left(\frac{\partial \delta n(t,x)}{\partial x}E(x) + n(t,x)\frac{\partial E(x)}{\partial x}\right) + g_n{}' - \frac{\delta n(t,x)}{\tau_n}$$

$$\boldsymbol{D}' = \frac{n\mu_n D_p + p\mu_p D_n}{n\mu_n + p\mu_p} = \frac{D_n D_p (p+n)}{D_n n + D_p p}\ [\mathrm{cm}^2/s]$$

$$\boldsymbol{\mu}' = \frac{\mu_n \mu_p (p-n)}{n\mu_n + p\mu_p}[\mathrm{cm}^2/V\cdot s]$$

Einstein's Relationship

$$\frac{D_n}{\mu_n} = \frac{D_p}{\mu_p} = \frac{kT}{q}[V]$$

Ambipolar Transport

Assume p-type Si and low-level injection

$$p(t,x) = p_0 + \delta p(t,x) \qquad \Rightarrow \qquad p(t,x) \cong p_0$$

$$n(t,x) = n_0 + \delta n(t,x) \qquad \qquad n(t,x) \cong \delta n(t,x)$$

$$\delta p(t,x) = \delta n(t,x) \qquad \qquad p(t,x) \gg n(t,x)$$

For **p-type Si** and low-level injection

Devide by pD_p

$$D' = \frac{D_n D_p (p+n)}{D_n n + D_p p} \quad \Rightarrow \quad D' = \frac{D_n (1 + n/p)}{1 + \frac{D_n}{D_p}\frac{n}{p}} \cong D_n$$

Devide by $p\mu_p$

$$\mu' = \frac{\mu_n \mu_p (p-n)}{n\mu_n + p\mu_p} \quad \Rightarrow \quad \mu' = \frac{\mu_n \left(1 - \frac{n}{p}\right)}{1 + \frac{n}{p}\frac{\mu_n}{\mu_p}} \cong \mu_n$$

$$\frac{\partial \delta N(t,x)}{\partial t} = \boldsymbol{D_n} \frac{\partial^2 \delta N(t,x)}{\partial x^2} + \boldsymbol{\mu_n} \frac{\partial \delta N(t,x)}{\partial x} E + g' - R'$$

Ambipolar transport

→ excess electron and hole move together as a pair → Single Mobility and Single Diffusion constant

→ minority carrier determines the ambipolar transport characteristics.

Ambipolar Transport Summary

Ambipolar transport

→ excess electron and hole move together as a pair → Single Mobility and Single Diffusion constant

→ minority carrier determines the ambipolar transport characteristics.

For **p-type Si** and low-level injection

$$\frac{\partial \delta N(t,x)}{\partial t} = \boldsymbol{D_n} \frac{\partial^2 \delta N(t,x)}{\partial x^2} + \boldsymbol{\mu_n} \frac{\partial \delta N(t,x)}{\partial x} E + g' - R'$$

For **n-type Si** and low-level injection

$$\frac{\partial \delta N(t,x)}{\partial t} = \boldsymbol{D_p} \frac{\partial^2 \delta N(t,x)}{\partial x^2} - \boldsymbol{\mu_p} \frac{\partial \delta N(t,x)}{\partial x} E + g' - R'$$

Ambipolar Transport

Additional assumptions

- No excess carrier generation $g' = 0$
- No external (applied) E-field $E = 0$
- Steady-state condition $\dfrac{dn}{dt} = \dfrac{dp}{dt} = 0$

⇨ Current is only generated by Diffusion and Recombination.

$$\frac{\delta N(x)}{\tau} = R'$$

<u>For p-type Si and low-level injection</u>

$$\overset{=0}{\frac{\partial \delta N(t,x)}{\partial t}} = D_n \frac{\partial^2 \delta N(t,x)}{\partial x^2} + \mu_n \frac{\partial \delta N(t,x)}{\partial x} \overset{=0}{E} + \overset{=0}{g'} - R' \Rightarrow 0 = D_n \frac{d^2 \delta N(x)}{dx^2} - R'$$

<u>For n-type Si and low-level injection</u>

$$\overset{=0}{\frac{\partial \delta N(t,x)}{\partial t}} = D_p \frac{\partial^2 \delta N(t,x)}{\partial x^2} - \mu_p \frac{\partial \delta N(t,x)}{\partial x} \overset{=0}{E} + \overset{=0}{g'} - R' \Rightarrow 0 = D_p \frac{d^2 \delta N(x)}{dx^2} - R'$$

Ambipolar Transport

For p-type

$$D_n \frac{d^2 \delta N(x)}{dx^2} = R'$$

$$\frac{\delta N(x)}{\tau_N} = R'$$

$$\Rightarrow$$

$$\frac{d^2 \delta N(x)}{dx^2} = \frac{\delta N(x)}{D_n \tau_n}$$

For n-type

$$D_p \frac{\partial^2 \delta N(x)}{dx^2} = R'$$

$$\frac{d^2 \delta N(x)}{dx^2} = \frac{\delta N(x)}{D_p \tau_p}$$

2nd order ODE

$$y'' = \frac{1}{L_N^2} y$$

$$\Downarrow$$

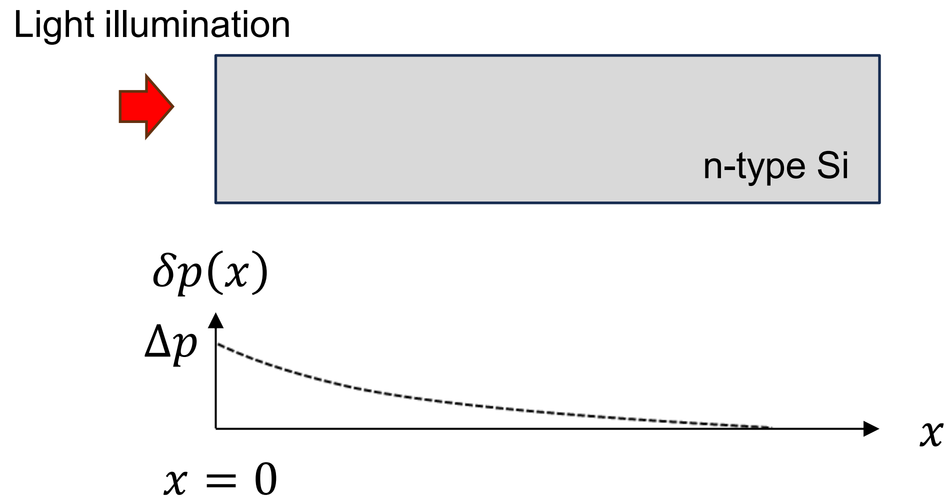
$$L_n = \sqrt{D_n \tau_n} \;\; or \;\; L_p = \sqrt{D_p \tau_p}$$

$$\delta N(x) = A e^{+\frac{x}{L_N}} + B e^{-\frac{x}{L_N}}$$

Ambipolar Transport (Example)

Additional assumptions

- No excess carrier generation
- No external (applied) E-field
- Steady-state condition



<u>For n-type</u>

$$\frac{d^2\delta N(x)}{dx^2} = \frac{\delta N(x)}{D_p\tau} \qquad L_p = \sqrt{D_p\tau_p} \quad \Rightarrow \quad \delta p(x) = Ae^{+\frac{x}{L}} + Be^{-\frac{x}{L}}$$

<u>Boundary condition</u>

$$x \to \infty, \qquad \delta p(x) \to 0 \quad \Rightarrow \quad A = 0$$

$$x = 0, \qquad \delta p(0) = \Delta p \quad \Rightarrow \quad B = \Delta p$$

$\Rightarrow$

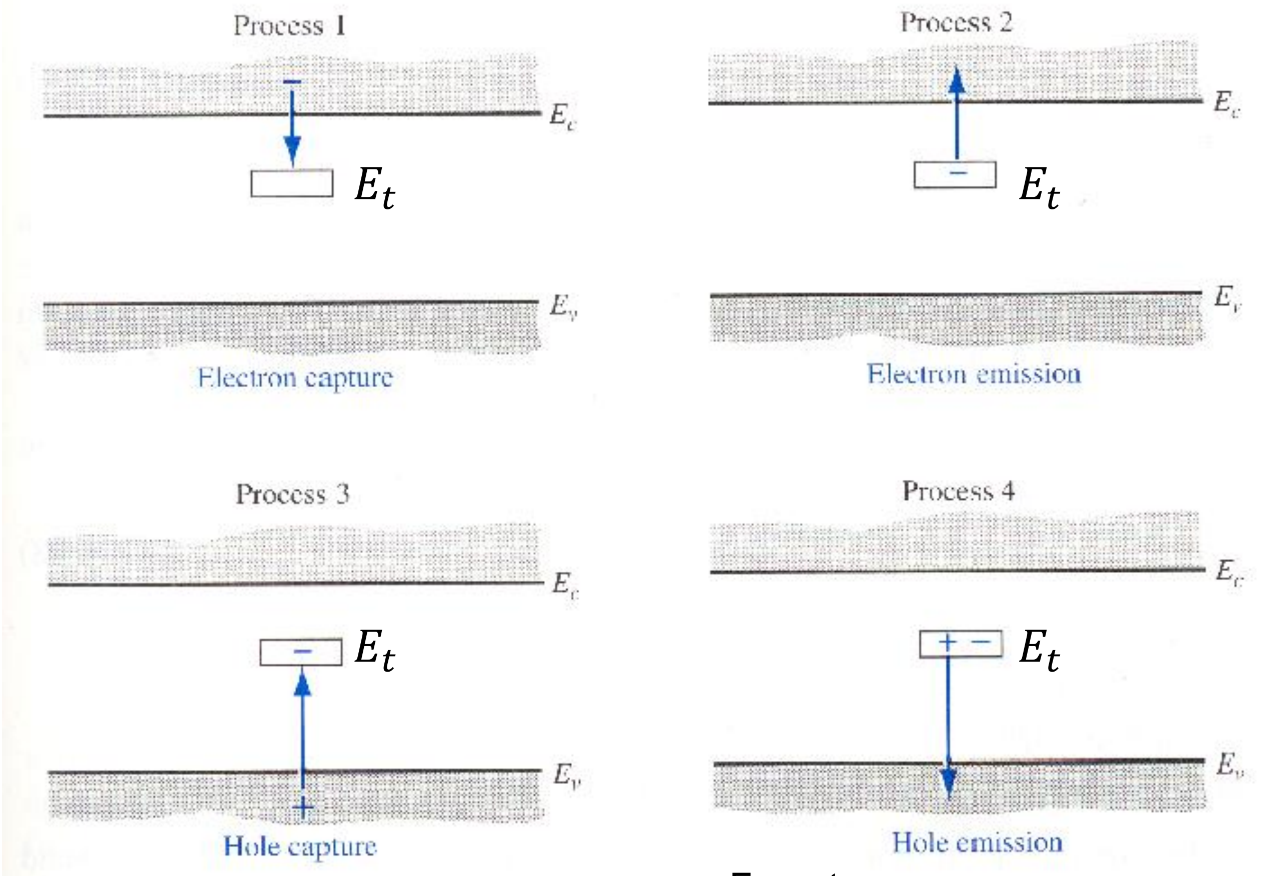
<u>Excess carrier concentration</u>

$$\delta p(x) = \Delta p e^{-\frac{x}{L_p}}$$

<u>Current density</u>

$$J_p(x) = J_{p|diff} = -qD_p\frac{dp(x)}{dx} = -qD_p\frac{d\delta p(x)}{dx} = \frac{qD_p\Delta p}{L_p}e^{-\frac{x}{L_p}} = q\frac{D_p}{L_p}\delta p(x)$$

Excess Carrier Lifetime



<u>For p-type</u>

$$\tau_{n0} = \frac{1}{C_n N_t}$$

<u>For n-type</u>

$$\tau_{p0} = \frac{1}{C_p N_t}$$

$$C_n(C_p) = constant\ proportional\ to\ electron(hole) - capture\ cross\ section$$

$$N_t = Total\ concenctraion\ of\ trapping\ centers$$